

Problem 7.8 An RHC-polarized wave with a modulus of 2 (V/m) is traveling in free space in the negative z -direction. Write the expression for the wave's electric field vector, given that the wavelength is 6 cm.

Solution:

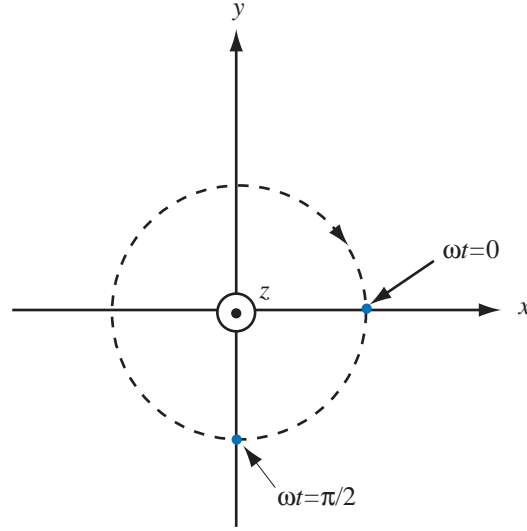


Figure P7.8: Locus of \mathbf{E} versus time.

For an RHC wave traveling in $-\hat{\mathbf{z}}$, let us try the following:

$$\mathbf{E} = \hat{\mathbf{x}}a \cos(\omega t + kz) + \hat{\mathbf{y}}a \sin(\omega t + kz).$$

Modulus $|\mathbf{E}| = \sqrt{a^2 + a^2} = a\sqrt{2} = 2$ (V/m). Hence,

$$a = \frac{2}{\sqrt{2}} = \sqrt{2}.$$

Next, we need to check the sign of the $\hat{\mathbf{y}}$ -component relative to that of the $\hat{\mathbf{x}}$ -component. We do this by examining the locus of \mathbf{E} versus t at $z = 0$: Since the wave is traveling along $-\hat{\mathbf{z}}$, when the thumb of the right hand is along $-\hat{\mathbf{z}}$ (into the page), the other four fingers point in the direction shown (clockwise as seen from above). Hence, we should reverse the sign of the $\hat{\mathbf{y}}$ -component:

$$\mathbf{E} = \hat{\mathbf{x}}\sqrt{2}\cos(\omega t + kz) - \hat{\mathbf{y}}\sqrt{2}\sin(\omega t + kz) \quad (\text{V/m})$$

with

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{6 \times 10^{-2}} = 104.72 \quad (\text{rad/m}),$$

and

$$\omega = kc = \frac{2\pi}{\lambda} \times 3 \times 10^8 = \pi \times 10^{10} \quad (\text{rad/s}).$$
