Problem 7.9 For a wave characterized by the electric field

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}\, a_x \cos(\omega t - kz) + \hat{\mathbf{y}}\, a_y \cos(\omega t - kz + \delta)$$

identify the polarization state, determine the polarization angles (γ, χ), and sketch the locus of $\mathbf{E}(0,t)$ for each of the following cases:

(a) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = 0$

(b) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = 180^\circ$

(c) $a_x = 3$ V/m, $a_y = 3$ V/m, and $\delta = 45^\circ$

(d) $a_x = 3$ V/m, $a_y = 4$ V/m, and $\delta = -135^\circ$

Solution:

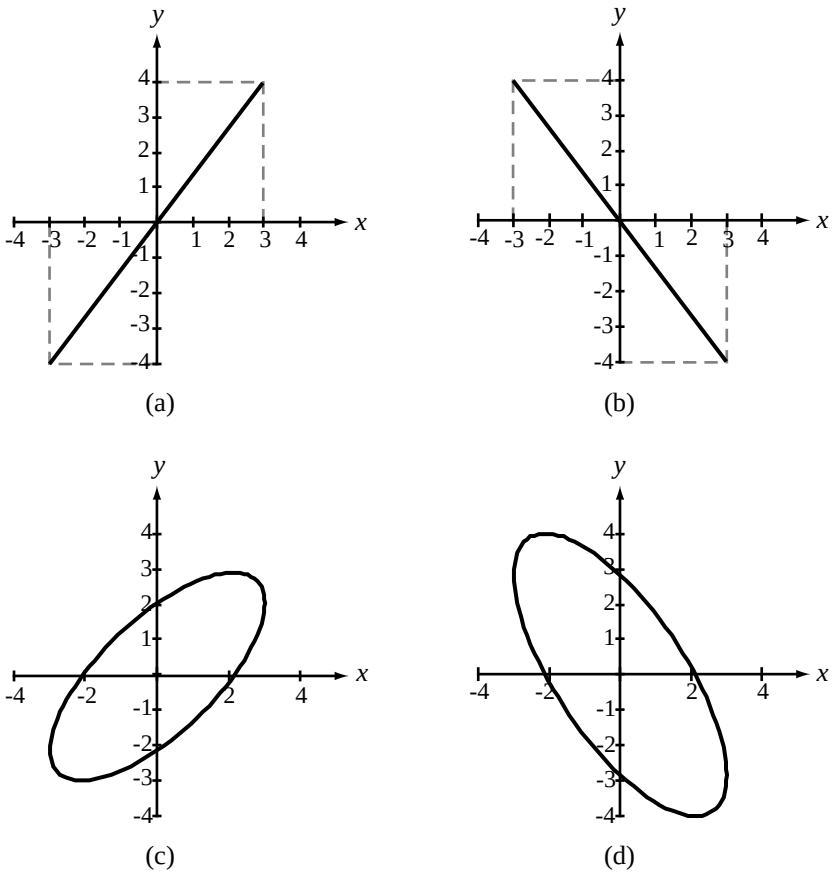

Figure P7.9: Plots of the locus of $\mathbf{E}(0,t)$.

$$\psi_0 = \tan^{-1}(a_y/a_x), \quad \text{[Eq. (7.60)]},$$
$$\tan 2\gamma = (\tan 2\psi_0)\cos\delta \quad \text{[Eq. (7.59a)]},$$
$$\sin 2\chi = (\sin 2\psi_0)\sin\delta \quad \text{[Eq. (7.59b)]}.$$

Case	a_x	a_y	δ	ψ_0	γ	χ	Polarization State
(a)	3	4	0	53.13°	53.13°	0	Linear
(b)	3	4	180°	53.13°	-53.13°	0	Linear
(c)	3	3	45°	45°	45°	22.5°	Left elliptical
(d)	3	4	-135°	53.13°	-56.2°	-21.37°	Right elliptical

(a) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) + \hat{\mathbf{y}}4\cos(\omega t - kz)$.
(b) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) - \hat{\mathbf{y}}4\cos(\omega t - kz)$.
(c) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) + \hat{\mathbf{y}}3\cos(\omega t - kz + 45^\circ)$.
(d) $\mathbf{E}(z,t) = \hat{\mathbf{x}}3\cos(\omega t - kz) + \hat{\mathbf{y}}4\cos(\omega t - kz - 135^\circ)$.