

**Problem 8.10** For the configuration shown in Fig. P8.9, use transmission-line equations (or the Smith chart) to calculate the input impedance at  $z = -d$  for  $\epsilon_{r1} = 1$ ,  $\epsilon_{r2} = 9$ ,  $\epsilon_{r3} = 4$ ,  $d = 1.2$  m, and  $f = 50$  MHz. Also determine the fraction of the incident average power density reflected by the structure. Assume all media are lossless and nonmagnetic.

**Solution:** In medium 2,

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_{r2}}} = \frac{c}{f\sqrt{\epsilon_{r2}}} = \frac{3 \times 10^8}{5 \times 10^7 \times 3} = 2 \text{ m}.$$

Hence,

$$\beta_2 = \frac{2\pi}{\lambda_2} = \pi \text{ rad/m}, \quad \beta_2 d = 1.2\pi \text{ rad}.$$

At  $z = -d$ , the input impedance of a transmission line with load impedance  $Z_L$  is given by Eq. (2.63) as

$$Z_{\text{in}}(-d) = Z_0 \left( \frac{Z_L + jZ_0 \tan \beta_2 d}{Z_0 + jZ_L \tan \beta_2 d} \right).$$

In the present case,  $Z_0 = \eta_2 = \eta_0/\sqrt{\epsilon_{r2}} = \eta_0/3$  and  $Z_L = \eta_3 = \eta_0/\sqrt{\epsilon_{r3}} = \eta_0/2$ , where  $\eta_0 = 120\pi \text{ } (\Omega)$ . Hence,

$$Z_{\text{in}}(-d) = \eta_2 \left( \frac{\eta_3 + j\eta_2 \tan \beta_2 d}{\eta_2 + j\eta_3 \tan \beta_2 d} \right) = \frac{\eta_0}{3} \left( \frac{\frac{1}{2} + j\left(\frac{1}{3}\right) \tan 1.2\pi}{\frac{1}{3} + j\left(\frac{1}{2}\right) \tan 1.2\pi} \right) = \eta_0(0.35 - j0.14).$$

At  $z = -d$ ,

$$\Gamma = \frac{Z_{\text{in}} - Z_1}{Z_{\text{in}} + Z_1} = \frac{\eta_0(0.35 - j0.14) - \eta_0}{\eta_0(0.35 - j0.14) + \eta_0} = 0.49e^{-j162.14^\circ}.$$

Fraction of incident power reflected by the structure is  $|\Gamma|^2 = |0.49|^2 = 0.24$ .

---