

Problem 8.11 Repeat Problem 8.10, but interchange ϵ_{r1} and ϵ_{r3} .

Solution: In medium 2,

$$\lambda = \frac{\lambda_0}{\sqrt{\epsilon_{r2}}} = \frac{c}{f\sqrt{\epsilon_{r2}}} = \frac{3 \times 10^8}{5 \times 10^7 \times 3} = 2 \text{ m.}$$

Hence,

$$\beta_2 = \frac{2\pi}{\lambda_2} = \pi \text{ rad/m,} \quad \beta_2 d = 1.2\pi \text{ rad.}$$

At $z = -d$, the input impedance of a transmission line with impedance Z_L is given as Eq. (2.63),

$$Z_{in}(-d) = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta d}{Z_0 + jZ_L \tan \beta d} \right).$$

In the present case, $Z_0 = \eta_2 = \eta_0 / \sqrt{\epsilon_{r2}} = \eta_0 / 3$, $Z_L = \eta_3 = \eta_0 / \sqrt{\epsilon_{r1}} = \eta_0$, where $\eta_0 = 120\pi (\Omega)$. Hence,

$$\begin{aligned} Z_{in}(-d) &= \eta_2 \left(\frac{\eta_3 + j\eta_2 \tan 1.2\pi}{\eta_2 + j\eta_3 \tan 1.2\pi} \right) \\ &= \frac{\eta_0}{3} \left(\frac{1 + (j/3) \tan 1.2\pi}{(1/3) + j \tan 1.2\pi} \right) \\ &= \eta_0 \left(\frac{1 + (j/3) \tan 1.2\pi}{1 + j3 \tan 1.2\pi} \right) = (0.266 - j0.337)\eta_0 = 0.43\eta_0 \angle -51.7^\circ. \end{aligned}$$

At $z = -d$,

$$\Gamma = \frac{Z_{in} - Z_1}{Z_{in} + Z_1} = \frac{0.43 \angle -51.7^\circ - \frac{1}{2}}{0.43 \angle -51.7^\circ + \frac{1}{2}} = 0.49 \angle -101.1^\circ.$$

Fraction of incident power reflected by structure is $|\Gamma|^2 = 0.24$.
