

Problem 8.15 A 5-MHz plane wave with electric field amplitude of 10 (V/m) is normally incident in air onto the plane surface of a semi-infinite conducting material with $\epsilon_r = 4$, $\mu_r = 1$, and $\sigma = 100$ (S/m). Determine the average power dissipated (lost) per unit cross-sectional area in a 2-mm penetration of the conducting medium.

Solution: For convenience, let us choose \mathbf{E}^i to be along $\hat{\mathbf{x}}$ and the incident direction to be $+\hat{\mathbf{z}}$. With

$$k_1 = \frac{\omega}{c} = \frac{2\pi \times 5 \times 10^6}{3 \times 10^8} = \frac{\pi}{30} \quad (\text{rad/m}),$$

we have

$$\mathbf{E}^i = \hat{\mathbf{x}} 10 \cos\left(\pi \times 10^7 t - \frac{\pi}{30} z\right) \quad (\text{V/m}),$$

$$\eta_1 = \eta_0 = 377 \, \Omega.$$

From Table 7-1,

$$\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega \epsilon_r \epsilon_0} = \frac{100 \times 36\pi}{\pi \times 10^7 \times 4 \times 10^{-9}} = 9 \times 10^4,$$

which makes the material a good conductor, for which

$$\alpha_2 = \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 5 \times 10^6 \times 4\pi \times 10^{-7} \times 100} = 44.43 \quad (\text{Np/m}),$$

$$\beta_2 = 44.43 \quad (\text{rad/m}),$$

$$\eta_{c2} = (1 + j) \frac{\alpha_2}{\sigma} = (1 + j) \frac{44.43}{100} = 0.44(1 + j) \, \Omega.$$

According to the expression for \mathbf{S}_{av_2} given in the answer to Exercise 8.3,

$$\mathbf{S}_{\text{av}_2} = \hat{\mathbf{z}} |\tau|^2 \frac{|E_0^i|^2}{2} e^{-2\alpha_2 z} \Re \left(\frac{1}{\eta_{c2}^*} \right).$$

The power lost is equal to the difference between \mathbf{S}_{av_2} at $z = 0$ and \mathbf{S}_{av_2} at $z = 2$ mm. Thus,

$$\begin{aligned} P' &= \text{power lost per unit cross-sectional area} \\ &= S_{\text{av}_2}(0) - S_{\text{av}_2}(z = 2 \text{ mm}) \\ &= |\tau|^2 \frac{|E_0^i|^2}{2} \Re \left(\frac{1}{\eta_{c2}^*} \right) [1 - e^{-2\alpha_2 z_1}] \end{aligned}$$

where $z_1 = 2$ mm.

$$\tau = 1 + \Gamma$$

$$= 1 + \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = 1 + \frac{0.44(1 + j) - 377}{0.44(1 + j) + 377} \approx 0.0023(1 + j) = 3.3 \times 10^{-3} e^{j45^\circ}.$$

$$\begin{aligned}\Re\left(\frac{1}{\eta_{c_2}^*}\right) &= \Re\left(\frac{1}{0.44(1+j)^*}\right) \\ &= \Re\left(\frac{1}{0.44(1-j)}\right) = \Re\left(\frac{1+j}{0.44 \times 2}\right) = \frac{1}{0.88} = 1.14,\end{aligned}$$

$$P' = (3.3 \times 10^{-3})^2 \frac{10^2}{2} \times 1.14 [1 - e^{-2 \times 44.43 \times 2 \times 10^{-3}}] = 1.01 \times 10^{-4} \quad (\text{W/m}^2).$$
