

Problem 8.16 A 0.5-MHz antenna carried by an airplane flying over the ocean surface generates a wave that approaches the water surface in the form of a normally incident plane wave with an electric-field amplitude of 3,000 (V/m). Seawater is characterized by $\epsilon_r = 72$, $\mu_r = 1$, and $\sigma = 4$ (S/m). The plane is trying to communicate a message to a submarine submerged at a depth d below the water surface. If the submarine's receiver requires a minimum signal amplitude of 0.01 ($\mu\text{V/m}$), what is the maximum depth d to which successful communication is still possible?

Solution: For sea water at 0.5 MHz,

$$\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon} = \frac{4 \times 36\pi}{2\pi \times 0.5 \times 10^6 \times 72 \times 10^{-9}} = 2000.$$

Hence, sea water is a good conductor, in which case we use the following expressions from Table 7-1:

$$\begin{aligned}\alpha_2 &= \sqrt{\pi f \mu \sigma} = \sqrt{\pi \times 0.5 \times 10^6 \times 4\pi \times 10^{-7} \times 4} = 2.81 \quad (\text{Np/m}), \\ \beta_2 &= 2.81 \quad (\text{rad/m}), \\ \eta_{c2} &= (1+j) \frac{\alpha_2}{\sigma} = (1+j) \frac{2.81}{4} = 0.7(1+j) \Omega, \\ \Gamma &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{0.7(1+j) - 377}{0.7(1+j) + 377} = (-0.9963 + j3.7 \times 10^{-3}), \\ \tau &= 1 + \Gamma = 5.24 \times 10^{-3} e^{j44.89^\circ}, \\ |E^t| &= |\tau E_0^i e^{-\alpha_2 d}|.\end{aligned}$$

We need to find the depth z at which $|E^t| = 0.01 \mu\text{V/m} = 10^{-8} \text{ V/m}$.

$$\begin{aligned}10^{-8} &= 5.24 \times 10^{-3} \times 3 \times 10^3 e^{-2.81 d}, \\ e^{-2.81 d} &= 6.36 \times 10^{-10}, \\ -2.81 d &= \ln(6.36 \times 10^{-10}) = -21.18,\end{aligned}$$

or

$$d = 7.54 \quad (\text{m}).$$
