8.17 A light ray is incident on a prism in air at an angle θ as shown in Fig. P8.17. The ray is refracted at the first surface and again at the second surface. In terms of the apex angle ϕ of the prism and its index of refraction n, determine the smallest value of θ for which the ray will emerge from the other side. Find this minimum θ for $n = 1.4$ and $\phi = 60°$.

Solution:

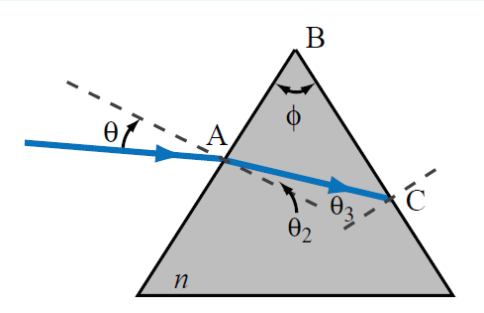

Figure P8.17 Prism of Problem 8.17.

For the beam to emerge at the second boundary, it is necessary that

$$\theta_3 < \theta_c,$$

where $\sin\theta_c = 1/n$. From the geometry of triangle ABC,

$$180° = \phi + (90° - \theta_2) + (90° - \theta_3),$$

or $\theta_2 = \phi - \theta_3$. At the first boundary, $\sin\theta = n\sin\theta_2$. Hence,

$$\sin\theta_{min} = n\sin(\phi - \theta_3) = n\sin\left(\phi - \sin^{-1}\left(\frac{1}{n}\right)\right),$$

or

$$\theta_{min} = \sin^{-1}\left[n\sin\left(\phi - \sin^{-1}\left(\frac{1}{n}\right)\right)\right].$$

For $n = 1.4$ and $\phi = 60°$,

$$\theta_{min} = \sin^{-1}\left[1.4\sin(60° - \sin^{-1}\left(\frac{1}{1.4}\right)\right] = 20.4°.$$