Problem 8.26 Equation (8.45) was derived for the case where the light incident upon the sending end of the optical fiber extends over the entire acceptance cone shown in Fig. 8-12(b). Suppose the incident light is constrained to a narrower range extending between normal incidence and θ', where $\theta' < \theta_a$.

(a) Obtain an expression for the maximum data rate f_p in terms of θ'.

(b) Evaluate f_p for the fiber of Example 8-5 when $\theta' = 5°$.

Solution:

(a) For $\theta_i = \theta'$,

$$\sin\theta_2 = \frac{1}{n_f}\sin\theta',$$

$$l_{max} = \frac{l}{\cos\theta_2} = \frac{l}{\sqrt{1-\sin^2\theta_2}} = \frac{l}{\sqrt{1-\left(\frac{\sin\theta'}{n_f}\right)^2}} = \frac{l n_f}{\sqrt{n_f^2-(\sin\theta')^2}},$$

$$t_{max} = \frac{l_{max}}{u_p} = \frac{l_{max} n_f}{c} = \frac{l n_f^2}{c\sqrt{n_f^2-(\sin\theta')^2}},$$

$$t_{min} = \frac{l}{u_p} = l\,\frac{n_f}{c},$$

$$\tau = \Delta t = t_{max} - t_{min} = l\,\frac{n_f}{c}\left[\frac{n_f}{\sqrt{n_f^2-(\sin\theta')^2}} - 1\right],$$

$$f_p = \frac{1}{2\tau} = \frac{c}{2 l n_f}\left[\frac{n_f}{\sqrt{n_f^2-(\sin\theta')^2}} - 1\right]^{-1} \quad \text{(bits/s)}.$$

(b) For:

$$n_f = 1.52,$$
$$\theta' = 5°,$$
$$l = 1 \text{ km},$$
$$c = 3\times 10^8 \text{ m/s},$$
$$f_p = 59.88 \quad \text{(Mb/s)}.$$