

**Problem 8.27** A plane wave in air with

$$\tilde{\mathbf{E}}^i = \hat{\mathbf{y}} 20e^{-j(3x+4z)} \quad (\text{V/m})$$

is incident upon the planar surface of a dielectric material, with  $\epsilon_r = 4$ , occupying the half-space  $z \geq 0$ . Determine:

- (a) The polarization of the incident wave.
- (b) The angle of incidence.
- (c) The time-domain expressions for the reflected electric and magnetic fields.
- (d) The time-domain expressions for the transmitted electric and magnetic fields.
- (e) The average power density carried by the wave in the dielectric medium.

**Solution:**

(a)  $\tilde{\mathbf{E}}^i = \hat{\mathbf{y}} 20e^{-j(3x+4z)} \text{ V/m}.$

Since  $\mathbf{E}^i$  is along  $\hat{\mathbf{y}}$ , which is perpendicular to the plane of incidence, the wave is perpendicularly polarized.

(b) From Eq. (8.48a), the argument of the exponential is

$$-jk_1(x \sin \theta_i + z \cos \theta_i) = -j(3x + 4z).$$

Hence,

$$k_1 \sin \theta_i = 3, \quad k_1 \cos \theta_i = 4,$$

from which we determine that

$$\tan \theta_i = \frac{3}{4} \quad \text{or} \quad \theta_i = 36.87^\circ,$$

and

$$k_1 = \sqrt{3^2 + 4^2} = 5 \quad (\text{rad/m}).$$

Also,

$$\omega = u_p k = ck = 3 \times 10^8 \times 5 = 1.5 \times 10^9 \quad (\text{rad/s}).$$

(c)

$$\eta_1 = \eta_0 = 377 \, \Omega,$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_{r2}}} = \frac{\eta_0}{2} = 188.5 \, \Omega,$$

$$\theta_t = \sin^{-1} \left[ \frac{\sin \theta_i}{\sqrt{\epsilon_{r2}}} \right] = \sin^{-1} \left[ \frac{\sin 36.87^\circ}{\sqrt{4}} \right] = 17.46^\circ,$$

$$\Gamma_{\perp} = \frac{\eta_2 \cos \theta_i - \eta_1 \cos \theta_t}{\eta_2 \cos \theta_i + \eta_1 \cos \theta_t} = -0.41,$$

$$\tau_{\perp} = 1 + \Gamma_{\perp} = 0.59.$$

In accordance with Eq. (8.49a), and using the relation  $E_0^r = \Gamma_{\perp} E_0^i$ ,

$$\begin{aligned}\tilde{\mathbf{E}}^r &= -\hat{\mathbf{y}} 8.2 e^{-j(3x-4z)}, \\ \tilde{\mathbf{H}}^r &= -(\hat{\mathbf{x}} \cos \theta_i + \hat{\mathbf{z}} \sin \theta_i) \frac{8.2}{\eta_0} e^{-j(3x-4z)},\end{aligned}$$

where we used the fact that  $\theta_i = \theta_t$  and the  $z$ -direction has been reversed.

$$\begin{aligned}\mathbf{E}^r &= \Re[\tilde{\mathbf{E}}^r e^{j\omega t}] = -\hat{\mathbf{y}} 8.2 \cos(1.5 \times 10^9 t - 3x + 4z) \quad (\text{V/m}), \\ \mathbf{H}^r &= -(\hat{\mathbf{x}} 17.4 + \hat{\mathbf{z}} 13.06) \cos(1.5 \times 10^9 t - 3x + 4z) \quad (\text{mA/m}).\end{aligned}$$

(d) In medium 2,

$$k_2 = k_1 \sqrt{\frac{\epsilon_2}{\epsilon_1}} = 5\sqrt{4} = 20 \quad (\text{rad/m}),$$

and

$$\theta_t = \sin^{-1} \left[ \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin \theta_i \right] = \sin^{-1} \left[ \frac{1}{2} \sin 36.87^\circ \right] = 17.46^\circ$$

and the exponent of  $\mathbf{E}^t$  and  $\mathbf{H}^t$  is

$$-jk_2(x \sin \theta_t + z \cos \theta_t) = -j10(x \sin 17.46^\circ + z \cos 17.46^\circ) = -j(3x + 9.54z).$$

Hence,

$$\begin{aligned}\tilde{\mathbf{E}}^t &= \hat{\mathbf{y}} 20 \times 0.59 e^{-j(3x+9.54z)}, \\ \tilde{\mathbf{H}}^t &= (-\hat{\mathbf{x}} \cos \theta_t + \hat{\mathbf{z}} \sin \theta_t) \frac{20 \times 0.59}{\eta_2} e^{-j(3x+9.54z)}, \\ \mathbf{E}^t &= \Re[\tilde{\mathbf{E}}^t e^{j\omega t}] = \hat{\mathbf{y}} 11.8 \cos(1.5 \times 10^9 t - 3x - 9.54z) \quad (\text{V/m}), \\ \mathbf{H}^t &= (-\hat{\mathbf{x}} \cos 17.46^\circ + \hat{\mathbf{z}} \sin 17.46^\circ) \frac{11.8}{188.5} \cos(1.5 \times 10^9 t - 3x - 9.54z) \\ &= (-\hat{\mathbf{x}} 59.72 + \hat{\mathbf{z}} 18.78) \cos(1.5 \times 10^9 t - 3x - 9.54z) \quad (\text{mA/m}).\end{aligned}$$

(e)

$$S_{\text{av}}^t = \frac{|E_0^t|^2}{2\eta_2} = \frac{(11.8)^2}{2 \times 188.5} = 0.36 \quad (\text{W/m}^2).$$


---