

Problem 8.32 A perpendicularly polarized wave in air is obliquely incident upon a planar glass–air interface at an incidence angle of 30° . The wave frequency is 600 THz ($1 \text{ THz} = 10^{12} \text{ Hz}$), which corresponds to green light, and the index of refraction of the glass is 1.6. If the electric field amplitude of the incident wave is 50 V/m, determine the following:

- (a) The reflection and transmission coefficients.
- (b) The instantaneous expressions for \mathbf{E} and \mathbf{H} in the glass medium.

Solution:

(a) For nonmagnetic materials, $(\epsilon_2/\epsilon_1) = (n_2/n_1)^2$. Using this relation in Eq. (8.60) gives

$$\Gamma_{\perp} = \frac{\cos \theta_i - \sqrt{(n_2/n_1)^2 - \sin^2 \theta_i}}{\cos \theta_i + \sqrt{(n_2/n_1)^2 - \sin^2 \theta_i}} = \frac{\cos 30^\circ - \sqrt{(1.6)^2 - \sin^2 30^\circ}}{\cos 30^\circ + \sqrt{(1.6)^2 - \sin^2 30^\circ}} = -0.27,$$

$$\tau_{\perp} = 1 + \Gamma_{\perp} = 1 - 0.27 = 0.73.$$

(b) In the glass medium,

$$\sin \theta_t = \frac{\sin \theta_i}{n_2} = \frac{\sin 30^\circ}{1.6} = 0.31,$$

or $\theta_t = 18.21^\circ$.

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \frac{\eta_0}{n_2} = \frac{120\pi}{1.6} = 75\pi = 235.62 \quad (\Omega),$$

$$k_2 = \frac{\omega}{u_p} = \frac{2\pi f}{c/n} = \frac{2\pi f n}{c} = \frac{2\pi \times 600 \times 10^{12} \times 1.6}{3 \times 10^8} = 6.4\pi \times 10^6 \text{ rad/m},$$

$$E_0^t = \tau_{\perp} E_0^i = 0.73 \times 50 = 36.5 \text{ V/m}.$$

From Eqs. (8.49c) and (8.49d),

$$\tilde{\mathbf{E}}_{\perp}^t = \hat{\mathbf{y}} E_0^t e^{-jk_2(x \sin \theta_t + z \cos \theta_t)},$$

$$\tilde{\mathbf{H}}_{\perp}^t = (-\hat{\mathbf{x}} \cos \theta_t + \hat{\mathbf{z}} \sin \theta_t) \frac{E_0^t}{\eta_2} e^{-jk_2(x \sin \theta_t + z \cos \theta_t)},$$

and the corresponding instantaneous expressions are:

$$\mathbf{E}_{\perp}^t(x, z, t) = \hat{\mathbf{y}} 36.5 \cos(\omega t - k_2 x \sin \theta_t - k_2 z \cos \theta_t) \quad (\text{V/m}),$$

$$\mathbf{H}_{\perp}^t(x, z, t) = (-\hat{\mathbf{x}} \cos \theta_t + \hat{\mathbf{z}} \sin \theta_t) 0.16 \cos(\omega t - k_2 x \sin \theta_t - k_2 z \cos \theta_t) \quad (\text{A/m}),$$

with $\omega = 2\pi \times 10^{15} \text{ rad/s}$ and $k_2 = 6.4\pi \times 10^6 \text{ rad/m}$.
