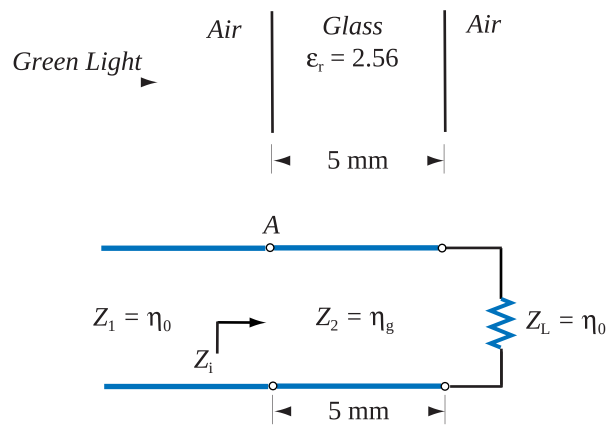
Problem 8.37 Consider a flat 5-mm–thick slab of glass with $\varepsilon_r = 2.56$.

(a) If a beam of green light ($\lambda_0 = 0.52$ μm) is normally incident upon one of the sides of the slab, what percentage of the incident power is reflected back by the glass?

(b) To eliminate reflections, it is desired to add a thin layer of antireflection coating material on each side of the glass. If you are at liberty to specify the thickness of the antireflection material as well as its relative permittivity, what would these specifications be?

Solution:



(a) Representing the wave propagation process by an equivalent transmission line model, the input impedance at the left-hand side of the air-glass interface is (from 2.63):

$$Z_i = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right)$$

For the glass,

$$Z_0 = \eta_g = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{\eta_0}{\sqrt{2.56}} = \frac{\eta_0}{1.6}$$

$$Z_L = \eta_0$$

$$\beta l = \frac{2\pi}{\lambda} l = \frac{2\pi}{\lambda_0} \sqrt{\varepsilon_r}\, l = \frac{2\pi}{0.52 \times 10^{-6}} \times \sqrt{2.56} \times 5 \times 10^{-3} = 30769.23\pi.$$

Subtracting the maximum possible multiples of 2π, namely 30768π, leaves a remainder of

$$\beta l = 1.23\pi \text{ rad.}$$

Hence,

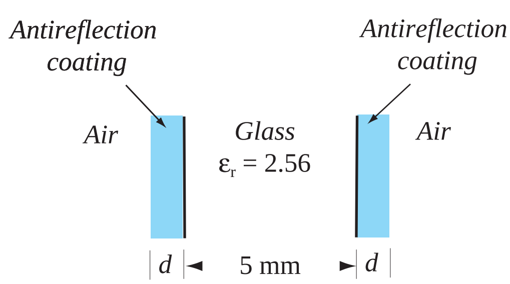
$$Z_{\rm i} = \frac{\eta_0}{1.6}\left(\frac{\eta_0 + j(\eta_0/1.6)\tan 1.23\pi}{(\eta_0/1.6) + j\eta_0 \tan 1.23\pi}\right)$$
$$= \left(\frac{1.6 + j\tan 1.23\pi}{1 + j1.6\tan 1.23\pi}\right)\frac{120\pi}{1.6}$$
$$= \left(\frac{1.6 + j0.882}{1 + j1.41}\right)\frac{120\pi}{1.6} = 249\angle{-25.8^\circ} = (224.2 - j108.4)\ \Omega.$$

With $Z_{\rm i}$ now representing the input impedance of the glass, the reflection coefficient at point A is:

$$\Gamma = \frac{Z_{\rm i} - \eta_0}{Z_{\rm i} + \eta_0}$$
$$= \frac{224.2 - j108.4 - 120\pi}{224.2 - j108.4 + 120\pi} = \frac{187.34\angle{-144.6^\circ}}{610.89\angle{-10.2^\circ}} = 0.3067\angle{-154.8^\circ}\ .$$

% of reflected power $= |\Gamma|^2 \times 100 = 9.4\%$.

(b) To avoid reflections, we can add a quarter-wave transformer on each side of the glass.



This requires that d be:

$$d = \frac{\lambda}{4} + 2n\lambda, \qquad n = 0, 1, 2, \ldots$$

where λ is the wavelength in that material; i.e., $\lambda = \lambda_0/\sqrt{\varepsilon_{\rm rc}}$, where $\varepsilon_{\rm rc}$ is the relative permittivity of the coating material. It is also required that $\eta_{\rm c}$ of the coating material be:

$$\eta_{\rm c}^2 = \eta_0 \eta_{\rm g}.$$

Thus

$$\frac{\eta_0^2}{\varepsilon_{\rm rc}} = \eta_0 \frac{\eta_0}{\sqrt{\varepsilon_{\rm r}}}\ ,$$

or

$$\varepsilon_{\mathrm{rc}} = \sqrt{\varepsilon_{\mathrm{r}}} = \sqrt{2.56} = 1.6.$$

Hence,

$$\begin{aligned}
\lambda &= \frac{\lambda_0}{\sqrt{\varepsilon_{\mathrm{rc}}}} = \frac{0.52\ \mu\mathrm{m}}{\sqrt{1.6}} = 0.411\ \mu\mathrm{m}, \\
d &= \frac{\lambda}{4} + 2n\lambda \\
&= (0.103 + 0.822n) \quad (\mu\mathrm{m}), \qquad n = 0, 1, 2, \ldots
\end{aligned}$$