

Problem 8.40 A TE wave propagating in a dielectric-filled waveguide of unknown permittivity has dimensions $a = 5$ cm and $b = 3$ cm. If the x -component of its electric field is given by

$$E_x = -36 \cos(40\pi x) \sin(100\pi y) \cdot \sin(2.4\pi \times 10^{10}t - 52.9\pi z), \quad (\text{V/m})$$

determine:

- (a) the mode number,
- (b) ϵ_r of the material in the guide,
- (c) the cutoff frequency, and
- (d) the expression for H_y .

Solution:

- (a) Comparison of the given expression with Eq. (8.110a) reveals that

$$\begin{aligned} \frac{m\pi}{a} &= 40\pi, & \text{hence } m &= 2 \\ \frac{n\pi}{b} &= 100\pi, & \text{hence } n &= 3. \end{aligned}$$

Mode is TE₂₃.

- (b) From $\sin(\omega t - \beta z)$, we deduce that

$$\omega = 2.4\pi \times 10^{10} \text{ rad/s}, \quad \beta = 52.9\pi \text{ rad/m}.$$

Using Eq. (8.105) to solve for ϵ_r , we have

$$\begin{aligned} \epsilon_r &= \frac{c^2}{\omega^2} \left[\beta^2 + \left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right] \\ &= 2.25. \end{aligned}$$

- (c)

$$\begin{aligned} u_{p0} &= \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8 \text{ m/s}. \\ f_{23} &= \frac{u_{p0}}{2} \sqrt{\left(\frac{2}{a} \right)^2 + \left(\frac{3}{b} \right)^2} \\ &= 10.77 \text{ GHz}. \end{aligned}$$

(d)

$$\begin{aligned} Z_{\text{TE}} = \frac{E_x}{H_y} &= \eta \left/ \sqrt{1 - (f_{23}/f)^2} \right. \\ &= \frac{377}{\sqrt{\epsilon_r}} \left/ \sqrt{1 - \left(\frac{10.77}{12} \right)^2} \right. = 569.9 \, \Omega. \end{aligned}$$

Hence,

$$\begin{aligned} H_y &= \frac{E_x}{Z_{\text{TE}}} \\ &= -0.063 \cos(40\pi x) \sin(100\pi y) \sin(2.4\pi \times 10^{10}t - 52.9\pi z) \quad (\text{A/m}). \end{aligned}$$
