**Problem 8.9** The three regions shown in Fig. P8.9 contain perfect dielectrics. For a wave in medium 1, incident normally upon the boundary at $z = -d$, what combination of $\varepsilon_{r_2}$ and $d$ produces no reflection? Express your answers in terms of $\varepsilon_{r_1}$, $\varepsilon_{r_3}$ and the oscillation frequency of the wave, $f$.

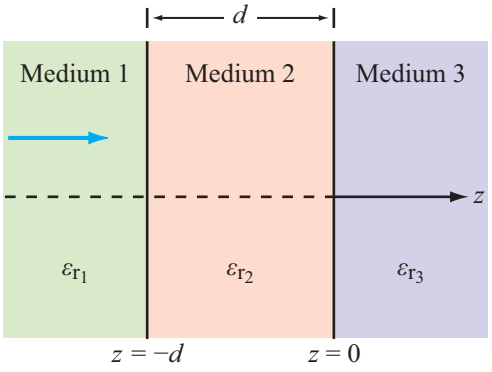


**Figure P8.9:** Dielectric layers for Problems 8.9 to 8.11.

**Solution:** By analogy with the transmission-line case, there will be no reflection at $z = -d$ if medium 2 acts as a quarter-wave transformer, which requires that

$$d = \frac{\lambda_2}{4}$$

and

$$\eta_2 = \sqrt{\eta_1 \eta_3}\,.$$

The second condition may be rewritten as

$$\frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \left[\frac{\eta_0}{\sqrt{\varepsilon_{r_1}}}\,\frac{\eta_0}{\sqrt{\varepsilon_{r_3}}}\right]^{1/2}, \qquad \text{or} \quad \varepsilon_{r_2} = \sqrt{\varepsilon_{r_1}\varepsilon_{r_3}}\,,$$

$$\lambda_2 = \frac{\lambda_0}{\sqrt{\varepsilon_{r_2}}} = \frac{c}{f\sqrt{\varepsilon_{r_2}}} = \frac{c}{f(\varepsilon_{r_1}\varepsilon_{r_3})^{1/4}}\,,$$

and

$$d = \frac{c}{4f(\varepsilon_{r_1}\varepsilon_{r_3})^{1/4}}\,.$$