

Problem 9.10 The normalized radiation intensity of a certain antenna is given by

$$F(\theta) = \exp(-20\theta^2) \quad \text{for } 0 \leq \theta \leq \pi$$

where θ is in radians. Determine:

- (a) The half-power beamwidth.
- (b) The pattern solid angle.
- (c) The antenna directivity.

Solution:

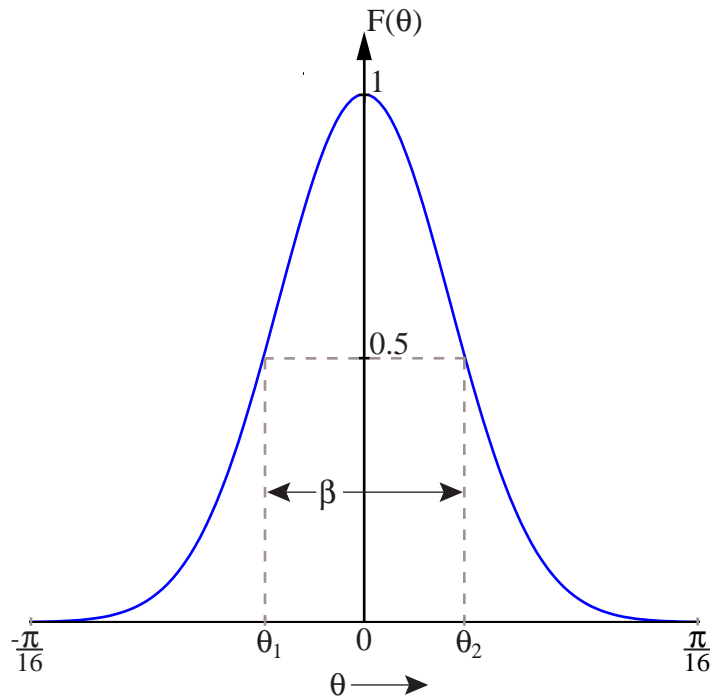


Figure P9.10: $F(\theta)$ versus θ .

(a) Since $F(\theta)$ is independent of ϕ , the beam is symmetrical about $z = 0$. Upon setting $F(\theta) = 0.5$, we have

$$\begin{aligned} F(\theta) &= \exp(-20\theta^2) = 0.5, \\ \ln[\exp(-20\theta^2)] &= \ln(0.5), \\ 20\theta^2 &= -0.693, \end{aligned}$$

$$\theta = \pm \left(\frac{0.693}{20} \right)^{1/2} = \pm 0.186 \text{ radians.}$$

Hence, $\beta = 2 \times 0.186 = 0.372 \text{ radians} = 21.31^\circ$.

(b) By Eq. (9.21),

$$\begin{aligned} \Omega_p &= \iint_{4\pi} F(\theta) \sin \theta \, d\theta \, d\phi \\ &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \exp(-20\theta^2) \sin \theta \, d\theta \, d\phi \\ &= 2\pi \int_0^{\pi} \exp(-20\theta^2) \sin \theta \, d\theta. \end{aligned}$$

Numerical evaluation yields

$$\Omega_p = 0.156 \text{ sr.}$$

(c)

$$D = \frac{4\pi}{\Omega_p} = \frac{4\pi}{0.156} = 80.55.$$
