Problem 9.22 A 150-MHz communication link consists of two vertical half-wave dipole antennas separated by 2 km. The antennas are lossless, the signal occupies a bandwidth of 3 MHz, the system noise temperature of the receiver is 600 K, and the desired signal-to-noise ratio is 17 dB. What transmitter power is required?

Solution: From Eq. (9.77), the receiver noise power is

$$P_{\rm n} = KT_{\rm sys}B = 1.38\times10^{-23}\times600\times3\times10^{6} = 2.48\times10^{-14}\ \text{W}.$$

For a signal to noise ratio $S_{\rm n} = 17\ \text{dB} = 50$, the received power must be at least

$$P_{\rm rec} = S_{\rm n}P_{\rm n} = 50(2.48\times10^{-14}\ \text{W}) = 1.24\times10^{-12}\ \text{W}.$$

Since the two antennas are half-wave dipoles, Eq. (9.47) states $D_{\rm t} = D_{\rm r} = 1.64$, and since the antennas are both lossless, $G_{\rm t} = D_{\rm t}$ and $G_{\rm r} = D_{\rm r}$. Since the operating frequency is $f = 150$ MHz, $\lambda = c/f = (3\times10^{8}\ \text{m/s})/(150\times10^{6}\ \text{Hz}) = 2$ m. Solving the Friis transmission formula (Eq. (9.75)) for the transmitted power:

$$P_{\rm t} = P_{\rm rec}\frac{(4\pi)^2R^2}{\lambda^2G_{\rm r}G_{\rm t}} = 1.24\times10^{-12}\frac{(4\pi)^2(2\times10^{3}\ \text{m})^2}{(2\ \text{m})^2(1.64)(1.64)} = 75\quad(\mu\text{W}).$$