

Problem 9.24 The configuration shown in Fig. P9.24 depicts two vertically oriented half-wave dipole antennas pointed towards each other, with both positioned on 100-m-tall towers separated by a distance of 5 km. If the transmit antenna is driven by a 50-MHz current with amplitude $I_0 = 2$ A, determine:

- The power received by the receive antenna in the absence of the surface. (Assume both antennas to be lossless.)
- The power received by the receive antenna after incorporating reflection by the ground surface, assuming the surface to be flat and to have $\epsilon_r = 9$ and conductivity $\sigma = 10^{-3}$ (S/m).

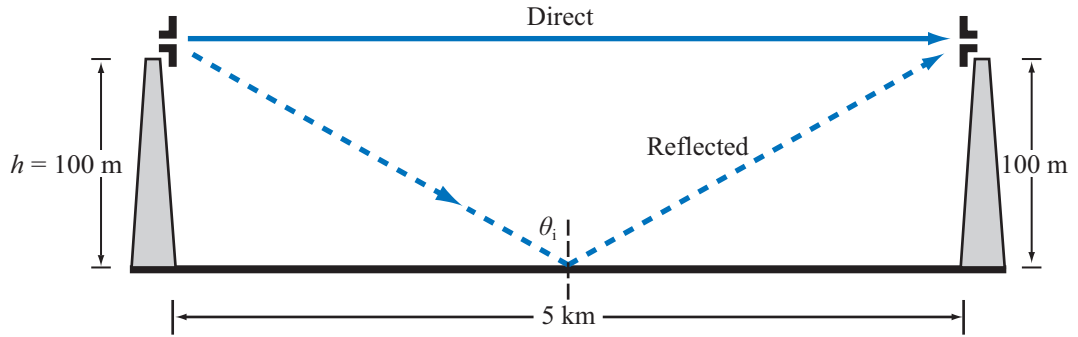


Figure P9.24: Problem 9.24.

Solution:

- Since both antennas are lossless,

$$P_{\text{rec}} = P_{\text{int}} = S_i A_{\text{er}}$$

where S_i is the incident power density and A_{er} is the effective area of the receive dipole. From Section 9-3,

$$S_i = S_0 = \frac{15I_0^2}{\pi R^2},$$

and from (9.64) and (9.47),

$$A_{\text{er}} = \frac{\lambda^2 D}{4\pi} = \frac{\lambda^2}{4\pi} \times 1.64 = \frac{1.64\lambda^2}{4\pi}.$$

Hence,

$$P_{\text{rec}} = \frac{15I_0^2}{\pi R^2} \times \frac{1.64\lambda^2}{4\pi} = 3.6 \times 10^{-6} \text{ W}.$$

(b) The electric field of the signal intercepted by the receive antenna now consists of a direct component, E_d , due to the directly transmitted signal, and a reflected component, E_r , due to the ground reflection. Since the power density S and the electric field E are related by

$$S = \frac{|E|^2}{2\eta_0},$$

it follows that

$$\begin{aligned} E_d &= \sqrt{2\eta_0 S_i} e^{-jkR} \\ &= \sqrt{2\eta_0 \times \frac{15I_0^2}{\pi R^2}} e^{-jkR} \\ &= \sqrt{\frac{30\eta_0}{\pi}} \frac{I_0}{R} e^{-jkR} \end{aligned}$$

where the phase of the signal is measured with respect to the location of the transmit antenna, and $k = 2\pi/\lambda$. Hence,

$$E_d = 0.024e^{-j120^\circ} \text{ (V/m)}.$$

The electric field of the reflected signal is similar in form except for the fact that R should be replaced with R' , where R' is the path length traveled by the reflected signal, and the electric field is modified by the reflection coefficient Γ . Thus,

$$E_r = \left(\sqrt{\frac{30\eta_0}{\pi}} \frac{I_0}{R'} e^{-jkR'} \right) \Gamma.$$

From the problem geometry

$$R' = 2\sqrt{(2.5 \times 10^3)^2 + (100)^2} = 5004.0 \text{ m}.$$

Since the dipole is vertically oriented, the electric field is parallel polarized. To calculate Γ , we first determine

$$\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega\epsilon_0\epsilon_r} = \frac{10^{-3}}{2\pi \times 50 \times 10^6 \times 8.85 \times 10^{-12} \times 9} = 0.04.$$

From Table 7-1,

$$\eta_c \approx \eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{\eta_0}{\sqrt{9}} = \frac{\eta_0}{3}.$$

From (8.66a),

$$\Gamma_{\parallel} = \frac{\eta_2 \cos \theta_t - \eta_1 \cos \theta_i}{\eta_2 \cos \theta_t + \eta_1 \cos \theta_i}$$

From the geometry,

$$\cos \theta_i = \frac{h}{(R'/2)} = \frac{100}{2502} = 0.04$$

$$\theta_i = 87.71^\circ$$

$$\theta_t = \sin^{-1} \left(\frac{\sin \theta_i}{\sqrt{\epsilon_r}} \right) = 19.46^\circ$$

$$\eta_1 = \eta_0 \text{ (air)}$$

$$\eta_2 = \eta = \frac{\eta_0}{3}.$$

Hence,

$$\Gamma_{\parallel} = \frac{(\eta_0/3) \times 0.94 - \eta_0 \times 0.04}{(\eta_0/3) \times 0.94 + \eta_0 \times 0.04} = 0.77.$$

The reflected electric field is

$$\begin{aligned} E_r &= \left(\sqrt{\frac{30\eta_0}{\pi}} \frac{I_0}{R'} e^{-jkR'} \right) \Gamma \\ &= 0.018e^{j0.6^\circ} \text{ (V/m)}. \end{aligned}$$

The total electric field is

$$\begin{aligned} E &= E_d + E_r \\ &= 0.024e^{-j120^\circ} + 0.018e^{j0.6^\circ} \\ &= 0.02e^{-j73.3^\circ} \text{ (V/m)}. \end{aligned}$$

The received power is

$$\begin{aligned} P_{\text{rec}} &= S_i A_{\text{er}} \\ &= \frac{|E|^2}{2\eta_0} \times \frac{1.64\lambda^2}{4\pi} \\ &= 2.5 \times 10^{-6} \text{ W}. \end{aligned}$$
