

**Problem 9.32** A two-element array consisting of two isotropic antennas separated by a distance  $d$  along the  $z$ -axis is placed in a coordinate system whose  $z$ -axis points eastward and whose  $x$ -axis points toward the zenith. If  $a_0$  and  $a_1$  are the amplitudes of the excitations of the antennas at  $z = 0$  and at  $z = d$ , respectively, and if  $\delta$  is the phase of the excitation of the antenna at  $z = d$  relative to that of the other antenna, find the array factor and plot the pattern in the  $x$ - $z$  plane for the following:

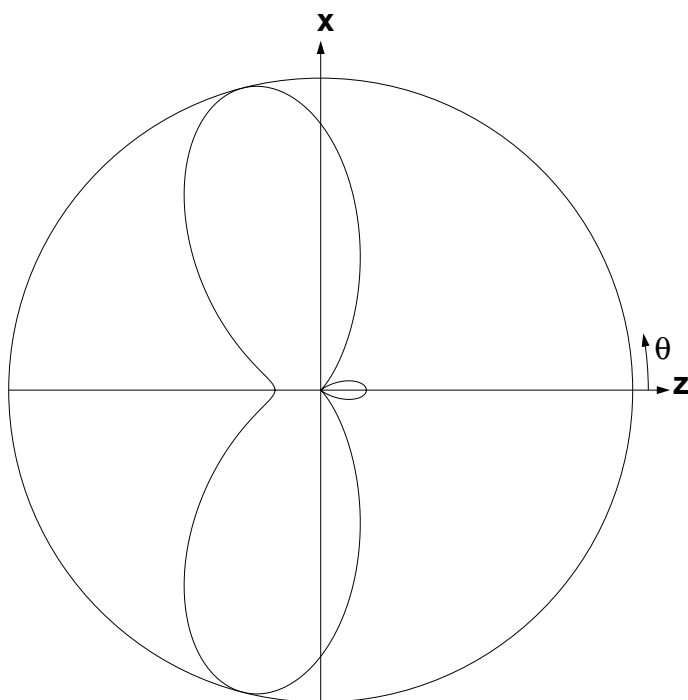
- (a)  $a_0 = a_1 = 1$ ,  $\delta = \pi/4$ , and  $d = \lambda/2$
- (b)  $a_0 = 1$ ,  $a_1 = 2$ ,  $\delta = 0$ , and  $d = \lambda$
- (c)  $a_0 = a_1 = 1$ ,  $\delta = -\pi/2$ , and  $d = \lambda/2$
- (d)  $a_0 = 1$ ,  $a_1 = 2$ ,  $\delta = \pi/4$ , and  $d = \lambda/2$
- (e)  $a_0 = 1$ ,  $a_1 = 2$ ,  $\delta = \pi/2$ , and  $d = \lambda/4$

**Solution:**

(a) Employing Eq. (9.110),

$$\begin{aligned}
 F_a(\theta) &= \left| \sum_{i=0}^1 a_i e^{j\psi_i} e^{j i k d \cos \theta} \right|^2 \\
 &= |1 + e^{j((2\pi/\lambda)(\lambda/2)\cos \theta + \pi/4)}|^2 \\
 &= |1 + e^{j(\pi \cos \theta + \pi/4)}|^2 = 4 \cos^2 \left[ \frac{\pi}{8} (4 \cos \theta + 1) \right].
 \end{aligned}$$

A plot of this array factor pattern is shown in Fig. 9-32(a).



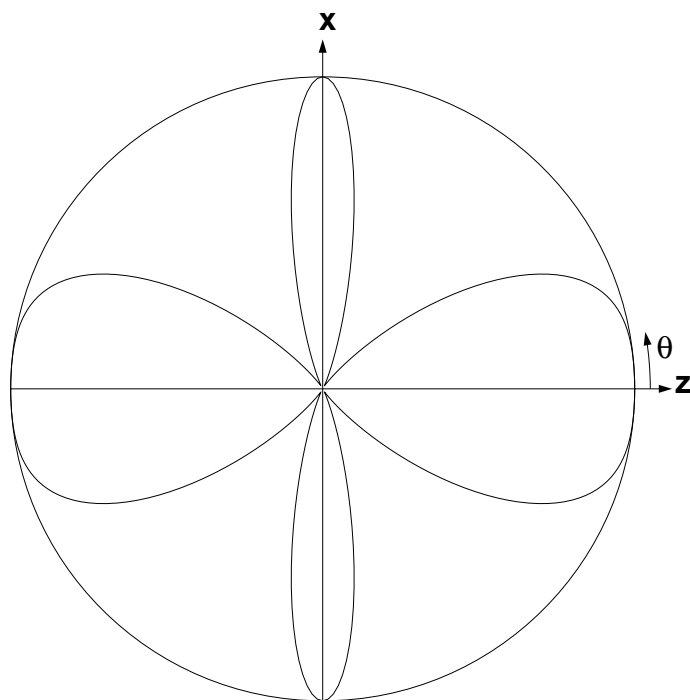
**Figure P9.32:** (a) Array factor in the elevation plane for Problem 9.32(a).

(b) Employing Eq. (9.110),

$$F_a(\theta) = \left| \sum_{i=0}^1 a_i e^{j\psi_i} e^{j i k d \cos \theta} \right|^2$$

$$= |1 + 2e^{j((2\pi/\lambda)\lambda \cos \theta + 0)}|^2 = |1 + 2e^{j2\pi \cos \theta}|^2 = 5 + 4 \cos(2\pi \cos \theta).$$

A plot of this array factor pattern is shown in Fig. 9-32(b).

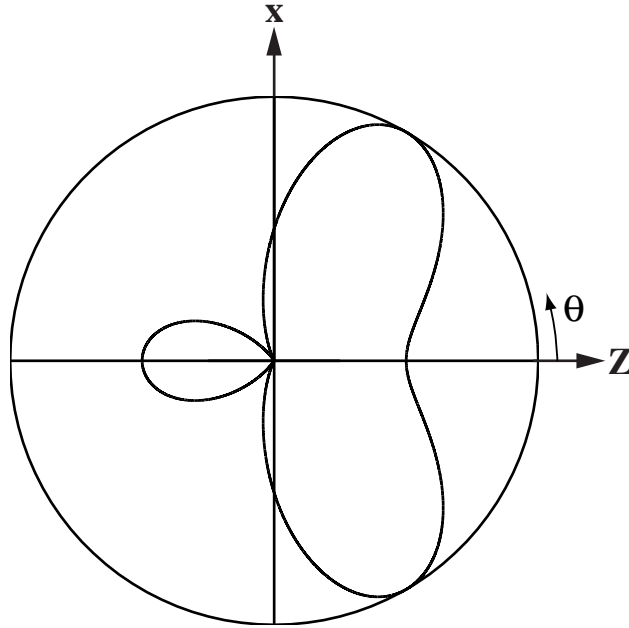


**Figure P9.32:** (b) Array factor in the elevation plane for Problem 9.32(b).

(c) Employing Eq. (9.110), and setting  $a_0 = a_1 = 1$ ,  $\psi = 0$ ,  $\psi_1 = \delta = -\pi/2$  and  $d = \lambda/2$ , we have

$$\begin{aligned}
 F_a(\theta) &= \left| \sum_{i=0}^1 a_i e^{j\psi_i} e^{j i k d \cos \theta} \right|^2 \\
 &= \left| 1 + e^{-j\pi/2} e^{j(2\pi/\lambda)(\lambda/2) \cos \theta} \right|^2 \\
 &= \left| 1 + e^{j(\pi \cos \theta - \pi/2)} \right|^2 \\
 &= 4 \cos^2 \left( \frac{\pi}{2} \cos \theta - \frac{\pi}{4} \right).
 \end{aligned}$$

A plot of the array factor is shown in Fig. 9-32(c).

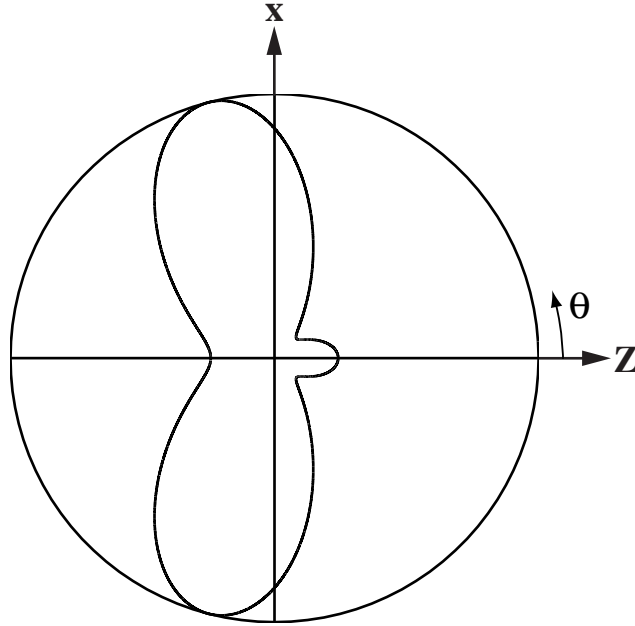


**Figure P9.32:** (c) Array factor in the elevation plane for Problem 9.32(c).

(d) Employing Eq. (9.110), and setting  $a_0 = 1$ ,  $a_1 = 2$ ,  $\psi_0 = 0$ ,  $\psi_1 = \delta = \pi/4$ , and  $d = \lambda/2$ , we have

$$\begin{aligned}
 F_a(\theta) &= \left| \sum_{i=0}^1 a_i e^{j\psi_i} e^{j i k d \cos \theta} \right|^2 \\
 &= \left| 1 + 2e^{j\pi/4} e^{j(2\pi/\lambda)(\lambda/2) \cos \theta} \right|^2 \\
 &= \left| 1 + 2e^{j(\pi \cos \theta + \pi/4)} \right|^2 \\
 &= 5 + 4 \cos \left( \pi \cos \theta + \frac{\pi}{4} \right).
 \end{aligned}$$

A plot of the array factor is shown in Fig. 9-32(d).

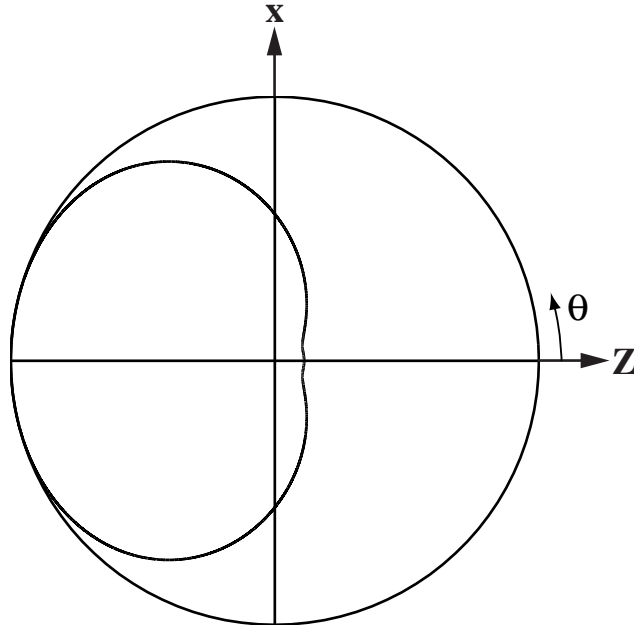


**Figure P9.32:** (d) Array factor in the elevation plane for Problem 9.32(d).

(e) Employing Eq. (9.110), and setting  $a_0 = 1$ ,  $a_1 = 2$ ,  $\psi_0 = 0$ ,  $\psi_1 = \delta = \pi/2$ , and  $d = \lambda/4$ , we have

$$\begin{aligned}
 F_a(\theta) &= \left| \sum_{i=0}^1 a_i e^{j\psi_i} e^{j i k d \cos \theta} \right|^2 \\
 &= \left| 1 + 2e^{j\pi/2} e^{j(2\pi/\lambda)(\lambda/4)\cos \theta} \right|^2 \\
 &= \left| 1 + 2e^{j(\pi \cos \theta + \pi)/2} \right|^2 \\
 &= 5 + 4 \cos \left( \frac{\pi}{2} \cos \theta + \frac{\pi}{2} \right) = 5 - 4 \sin \left( \frac{\pi}{2} \cos \theta \right).
 \end{aligned}$$

A plot of the array factor is shown in Fig. 9-32(e).



**Figure P9.32:** (e) Array factor in the elevation plane for Problem 9.32(e).