

Problem 9.37 A five-element equally spaced linear array with $d = \lambda/2$ is excited with uniform phase and an amplitude distribution given by the binomial distribution

$$a_i = \frac{(N-1)!}{i!(N-i-1)!}, \quad i = 0, 1, \dots, (N-1),$$

where N is the number of elements. Develop an expression for the array factor.

Solution: Using the given formula,

$$\begin{aligned} a_0 &= \frac{(5-1)!}{0!4!} = 1 \quad (\text{note that } 0! = 1) \\ a_1 &= \frac{4!}{1!3!} = 4 \\ a_2 &= \frac{4!}{2!2!} = 6 \\ a_3 &= \frac{4!}{3!1!} = 4 \\ a_4 &= \frac{4!}{0!4!} = 1 \end{aligned}$$

Application of (9.113) leads to:

$$\begin{aligned} F_a(\gamma) &= \left| \sum_{i=0}^{N-1} a_i e^{ji\gamma} \right|^2, \quad \gamma = \frac{2\pi d}{\lambda} \cos \theta \\ &= |1 + 4e^{j\gamma} + 6e^{j2\gamma} + 4e^{j3\gamma} + e^{j4\gamma}|^2 \\ &= |e^{j2\gamma}(e^{-j2\gamma} + 4e^{-j\gamma} + 6 + 4e^{j\gamma} + e^{j2\gamma})|^2 \\ &= (6 + 8\cos \gamma + 2\cos 2\gamma)^2. \end{aligned}$$

With $d = \lambda/2$, $\gamma = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos \theta = \pi \cos \theta$,

$$F_a(\theta) = [6 + 8\cos(\pi \cos \theta) + 2\cos(2\pi \cos \theta)]^2.$$
