Problem 9.38 A three-element linear array of isotropic sources aligned along the z-axis has an interelement spacing of $\lambda/4$ (Fig. P9.38). The amplitude excitation of the center element is twice that of the bottom and top elements, and the phases are $-\pi/2$ for the bottom element and $\pi/2$ for the top element, relative to that of the center element. Determine the array factor and plot it in the elevation plane.

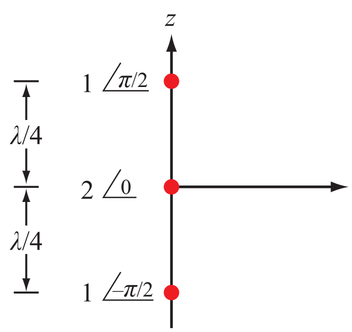

Figure P9.38: Three-element array of Problem 9.38.

Solution: From Eq. (9.110),

$$
\begin{aligned}
F_{\mathrm{a}}(\theta) &= \left|\sum_{i=0}^{2} a_i e^{j\psi_i} e^{jikd\cos\theta}\right|^2 \\
&= \left|a_0 e^{j\psi_0} + a_1 e^{j\psi_1} e^{jkd\cos\theta} + a_2 e^{j\psi_2} e^{j2kd\cos\theta}\right|^2 \\
&= \left|e^{j(\psi_1-\pi/2)} + 2e^{j\psi_1} e^{j(2\pi/\lambda)(\lambda/4)\cos\theta} + e^{j(\psi_1+\pi/2)} e^{j2(2\pi/\lambda)(\lambda/4)\cos\theta}\right|^2 \\
&= \left|e^{j\psi_1} e^{j(\pi/2)\cos\theta}\right|^2 \left|e^{-j\pi/2} e^{-j(\pi/2)\cos\theta} + 2 + e^{j\pi/2} e^{j(\pi/2)\cos\theta}\right|^2 \\
&= 4\left(1+\cos\left(\tfrac{1}{2}\pi(1+\cos\theta)\right)\right)^2, \\
F_{\mathrm{an}}(\theta) &= \tfrac{1}{4}\left(1+\cos\left(\tfrac{1}{2}\pi(1+\cos\theta)\right)\right)^2.
\end{aligned}
$$

This normalized array factor is shown in Fig. 9.38(b).

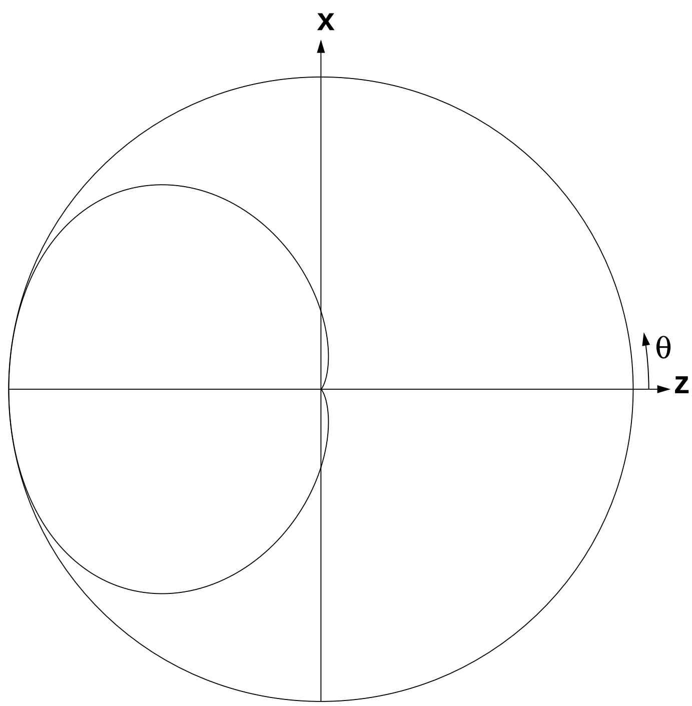

Figure P9.38: (b) Normalized array pattern of the 3-element array of Problem 9.38.