

1.15 A laser beam traveling through fog was observed to have an intensity of $1 \text{ } (\mu\text{W}/\text{m}^2)$ at a distance of 2 m from the laser gun and an intensity of $0.2 \text{ } (\mu\text{W}/\text{m}^2)$ at a distance of 3 m. Given that the intensity of an electromagnetic wave is proportional to the square of its electric-field amplitude, find the attenuation constant α of fog.

Solution: If the electric field is of the form

$$E(x, t) = E_0 e^{-\alpha x} \cos(\omega t - \beta x),$$

then the intensity must have a form

$$\begin{aligned} I(x, t) &\approx [E_0 e^{-\alpha x} \cos(\omega t - \beta x)]^2 \\ &\approx E_0^2 e^{-2\alpha x} \cos^2(\omega t - \beta x) \end{aligned}$$

or

$$I(x, t) = I_0 e^{-2\alpha x} \cos^2(\omega t - \beta x)$$

where we define $I_0 \approx E_0^2$. We observe that the magnitude of the intensity varies as $I_0 e^{-2\alpha x}$. Hence,

$$\begin{aligned} \text{at } x = 2 \text{ m,} \quad I_0 e^{-4\alpha} &= 1 \times 10^{-6} \quad (\text{W}/\text{m}^2), \\ \text{at } x = 3 \text{ m,} \quad I_0 e^{-6\alpha} &= 0.2 \times 10^{-6} \quad (\text{W}/\text{m}^2). \end{aligned}$$

$$\begin{aligned} \frac{I_0 e^{-4\alpha}}{I_0 e^{-6\alpha}} &= \frac{10^{-6}}{0.2 \times 10^{-6}} = 5 \\ e^{-4\alpha} \cdot e^{6\alpha} &= e^{2\alpha} = 5 \\ \alpha &= 0.8 \quad (\text{NP}/\text{m}). \end{aligned}$$
