

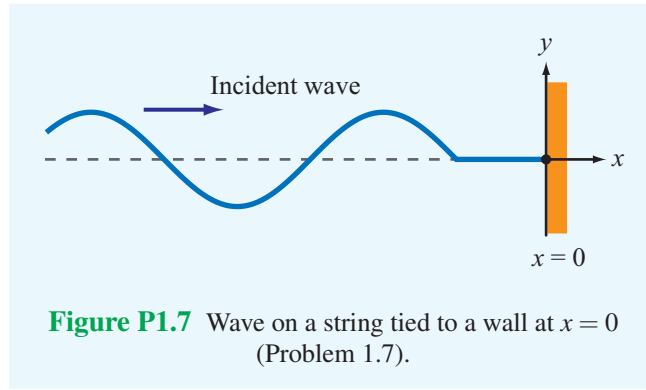
1.7 A wave traveling along a string in the $+x$ direction is given by

$$y_1(x, t) = A \cos(\omega t - \beta x),$$

where $x = 0$ is the end of the string, which is tied rigidly to a wall, as shown in Fig. P1.7. When wave $y_1(x, t)$ arrives at the wall, a reflected wave $y_2(x, t)$ is generated. Hence, at any location on the string, the vertical displacement y_s is the sum of the incident and reflected waves:

$$y_s(x, t) = y_1(x, t) + y_2(x, t).$$

- (a) Write an expression for $y_2(x, t)$, keeping in mind its direction of travel and the fact that the end of the string cannot move.
- (b) Generate plots of $y_1(x, t)$, $y_2(x, t)$ and $y_s(x, t)$ versus x over the range $-2\lambda \leq x \leq 0$ at $\omega t = \pi/4$ and at $\omega t = \pi/2$.



Solution:

(a) Since wave $y_2(x, t)$ was caused by wave $y_1(x, t)$, the two waves must have the same angular frequency ω , and since $y_2(x, t)$ is traveling on the same string as $y_1(x, t)$, the two waves must have the same phase constant β . Hence, with its direction being in the negative x -direction, $y_2(x, t)$ is given by the general form

$$y_2(x, t) = B \cos(\omega t + \beta x + \phi_0), \quad (1)$$

where B and ϕ_0 are yet-to-be-determined constants. The total displacement is

$$y_s(x, t) = y_1(x, t) + y_2(x, t) = A \cos(\omega t - \beta x) + B \cos(\omega t + \beta x + \phi_0).$$

Since the string cannot move at $x = 0$, the point at which it is attached to the wall, $y_s(0, t) = 0$ for all t . Thus,

$$y_s(0, t) = A \cos \omega t + B \cos(\omega t + \phi_0) = 0. \quad (2)$$

(i) Easy Solution: The physics of the problem suggests that a possible solution for (2) is $B = -A$ and $\phi_0 = 0$, in which case we have

$$y_2(x, t) = -A \cos(\omega t + \beta x). \quad (3)$$

(ii) Rigorous Solution: By expanding the second term in (2), we have

$$A \cos \omega t + B(\cos \omega t \cos \phi_0 - \sin \omega t \sin \phi_0) = 0,$$

or

$$(A + B \cos \phi_0) \cos \omega t - (B \sin \phi_0) \sin \omega t = 0. \quad (4)$$

This equation has to be satisfied for all values of t . At $t = 0$, it gives

$$A + B \cos \phi_0 = 0, \quad (5)$$

and at $\omega t = \pi/2$, (4) gives

$$B \sin \phi_0 = 0. \quad (6)$$

Equations (5) and (6) can be satisfied simultaneously only if

$$A = B = 0 \quad (7)$$

or

$$A = -B \quad \text{and} \quad \phi_0 = 0. \quad (8)$$

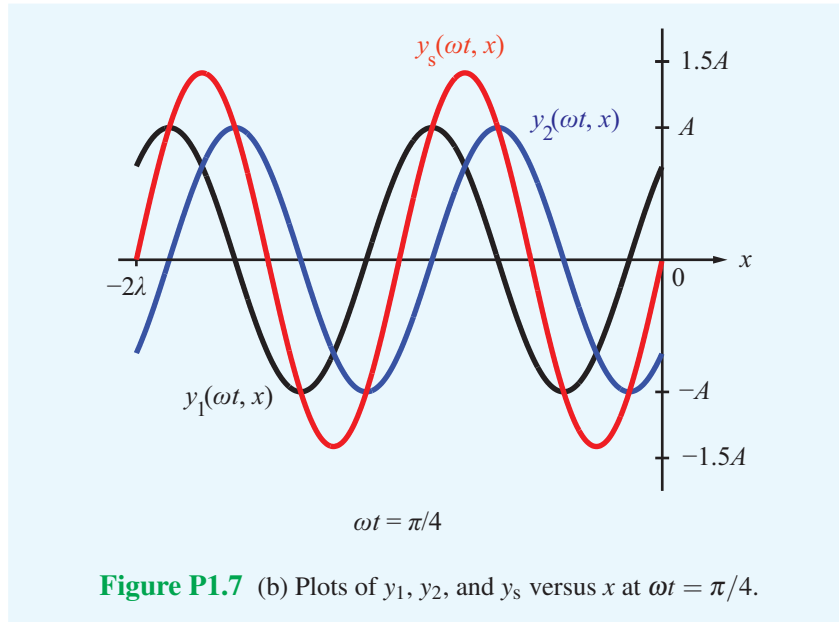
Clearly (7) is not an acceptable solution because it means that $y_1(x, t) = 0$, which is contrary to the statement of the problem. The solution given by (8) leads to (3).

(b) At $\omega t = \pi/4$,

$$y_1(x, t) = A \cos(\pi/4 - \beta x) = A \cos\left(\frac{\pi}{4} - \frac{2\pi x}{\lambda}\right),$$

$$y_2(x, t) = -A \cos(\omega t + \beta x) = -A \cos\left(\frac{\pi}{4} + \frac{2\pi x}{\lambda}\right).$$

Plots of y_1 , y_2 , and y_3 are shown in Fig. P1.7(b).



At $\omega t = \pi/2$,

$$y_1(x, t) = A \cos(\pi/2 - \beta x) = A \sin \beta x = A \sin \frac{2\pi x}{\lambda},$$

$$y_2(x, t) = -A \cos(\pi/2 + \beta x) = A \sin \beta x = A \sin \frac{2\pi x}{\lambda}.$$

Plots of y_1 , y_2 , and y_3 are shown in Fig. P1.7(c).

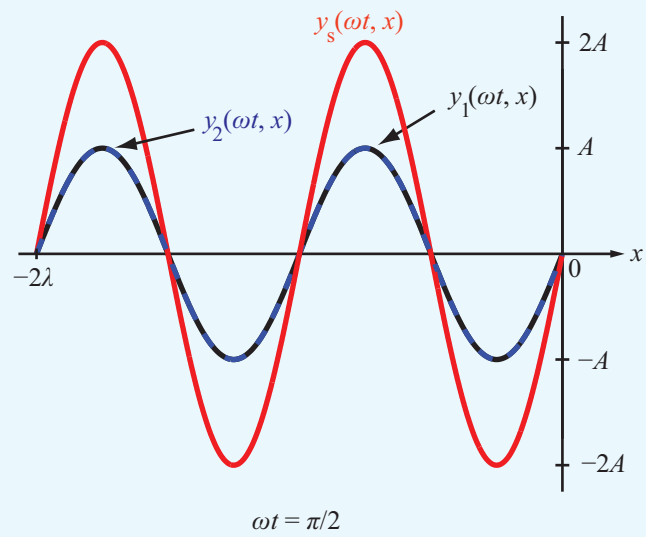


Figure P1.7 (c) Plots of y_1 , y_2 , and y_s versus x at $\omega t = \pi/2$.