

**1.9** Give expressions for  $y(x, t)$  for a sinusoidal wave traveling along a string in the negative  $x$ -direction, given that  $y_{\max} = 40$  cm,  $\lambda = 30$  cm,  $f = 10$  Hz, and

(a)  $y(x, 0) = 0$  at  $x = 0$ ,

(b)  $y(x, 0) = 0$  at  $x = 3.75$  cm.

**Solution:** For a wave traveling in the negative  $x$ -direction, we use Eq. (1.17) with  $\omega = 2\pi f = 20\pi$  (rad/s),  $\beta = 2\pi/\lambda = 2\pi/0.3 = 20\pi/3$  (rad/s),  $A = 40$  cm, and  $x$  assigned a positive sign:

$$y(x, t) = 40 \cos \left( 20\pi t + \frac{20\pi}{3}x + \phi_0 \right) \quad (\text{cm}),$$

with  $x$  in meters.

(a)  $y(0, 0) = 0 = 40 \cos \phi_0$ . Hence,  $\phi_0 = \pm\pi/2$ , and

$$\begin{aligned} y(x, t) &= 40 \cos \left( 20\pi t + \frac{20\pi}{3}x \pm \frac{\pi}{2} \right) \\ &= \begin{cases} -40 \sin \left( 20\pi t + \frac{20\pi}{3}x \right) \text{ (cm), if } \phi_0 = \pi/2, \\ 40 \sin \left( 20\pi t + \frac{20\pi}{3}x \right) \text{ (cm), if } \phi_0 = -\pi/2. \end{cases} \end{aligned}$$

(b) At  $x = 3.75$  cm  $= 3.75 \times 10^{-2}$  m,  $y = 0 = 40 \cos(\pi/4 + \phi_0)$ . Hence,  $\phi_0 = \pi/4$  or  $5\pi/4$ , and

$$y(x, t) = \begin{cases} 40 \cos \left( 20\pi t + \frac{20\pi}{3}x + \frac{\pi}{4} \right) \text{ (cm), if } \phi_0 = \pi/4, \\ 40 \cos \left( 20\pi t + \frac{20\pi}{3}x + \frac{5\pi}{4} \right) \text{ (cm), if } \phi_0 = 5\pi/4. \end{cases}$$


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