2.16 A transmission line operating at 125 MHz has $Z_0 = 40\ \Omega$, $\alpha = 0.02$ (Np/m), and $\beta = 0.75$ rad/m. Find the line parameters R', L', G', and C'.

Solution: Given an arbitrary transmission line, $f = 125$ MHz, $Z_0 = 40\ \Omega$, $\alpha = 0.02$ Np/m, and $\beta = 0.75$ rad/m. Since Z_0 is real and $\alpha \neq 0$, the line is distortionless. From Problem 2.13, $\beta = \omega\sqrt{L'C'}$ and $Z_0 = \sqrt{L'/C'}$, therefore,

$$L' = \frac{\beta Z_0}{\omega} = \frac{0.75 \times 40}{2\pi \times 125 \times 10^6} = 38.2 \text{ nH/m}.$$

Then, from $Z_0 = \sqrt{L'/C'}$,

$$C' = \frac{L'}{Z_0^2} = \frac{38.2 \text{ nH/m}}{40^2} = 23.9 \text{ pF/m}.$$

From $\alpha = \sqrt{R'G'}$ and $R'C' = L'G'$,

$$R' = \sqrt{R'G'}\sqrt{\frac{R'}{G'}} = \sqrt{R'G'}\sqrt{\frac{L'}{C'}} = \alpha Z_0 = 0.02 \text{ Np/m} \times 40\ \Omega = 0.6\ \Omega/\text{m}$$

and

$$G' = \frac{\alpha^2}{R'} = \frac{(0.02 \text{ Np/m})^2}{0.8\ \Omega/\text{m}} = 0.5 \text{ mS/m}.$$