2.21 On a 150 Ω lossless transmission line, the following observations were noted: distance of first voltage minimum from the load = 3 cm; distance of first voltage maximum from the load = 9 cm; $S = 3$. Find Z_L.

Solution: Distance between a minimum and an adjacent maximum = $\lambda/4$. Hence,

$$9\text{ cm} - 3\text{ cm} = 6\text{ cm} = \lambda/4,$$

or $\lambda = 24$ cm. Accordingly, the first voltage minimum is at $d_{\min} = 3\text{ cm} = \frac{\lambda}{8}$. Application of Eq. (2.71) with $n = 0$ gives

$$\theta_r - 2 \times \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = -\pi,$$

which gives $\theta_r = -\pi/2$.

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{2}{4} = 0.5.$$

Hence, $\Gamma = 0.5\, e^{-j\pi/2} = -j0.5$.

Finally,

$$Z_L = Z_0 \left[\frac{1+\Gamma}{1-\Gamma}\right] = 150 \left[\frac{1-j0.5}{1+j0.5}\right] = (90 - j120)\ \Omega.$$