

2.3 Show that the transmission line model shown in Fig. P2.3 yields the same telegrapher's equations given by Eqs. (2.14) and (2.16).

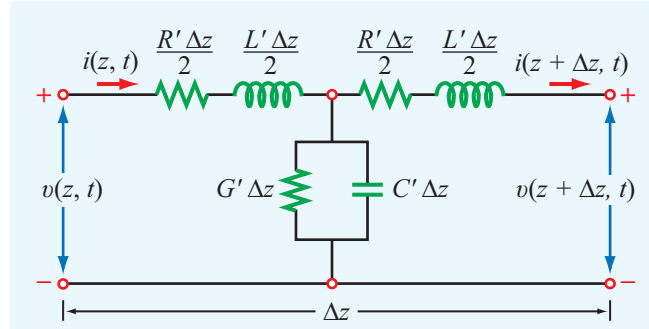


Figure P2.3 Transmission line model.

Solution: The voltage at the central upper node is the same whether it is calculated from the left port or the right port:

$$\begin{aligned} v(z + \tfrac{1}{2}\Delta z, t) &= v(z, t) - \tfrac{1}{2}R'\Delta z i(z, t) - \tfrac{1}{2}L'\Delta z \frac{\partial}{\partial t} i(z, t) \\ &= v(z + \Delta z, t) + \tfrac{1}{2}R'\Delta z i(z + \Delta z, t) + \tfrac{1}{2}L'\Delta z \frac{\partial}{\partial t} i(z + \Delta z, t). \end{aligned}$$

Recognizing that the current through the $G' \parallel C'$ branch is $i(z, t) - i(z + \Delta z, t)$ (from Kirchhoff's current law), we can conclude that

$$i(z, t) - i(z + \Delta z, t) = G'\Delta z v(z + \tfrac{1}{2}\Delta z, t) + C'\Delta z \frac{\partial}{\partial t} v(z + \tfrac{1}{2}\Delta z, t).$$

From both of these equations, the proof is completed by following the steps outlined in the text, ie. rearranging terms, dividing by Δz , and taking the limit as $\Delta z \rightarrow 0$.
