

2.44 For the circuit shown in Fig. P2.44, calculate the average incident power, the average reflected power, and the average power transmitted into the infinite $100\text{-}\Omega$ line. The $\lambda/2$ line is lossless and the infinitely long line is slightly lossy. (Hint: The input impedance of an infinitely long line is equal to its characteristic impedance so long as $\alpha \neq 0$.)

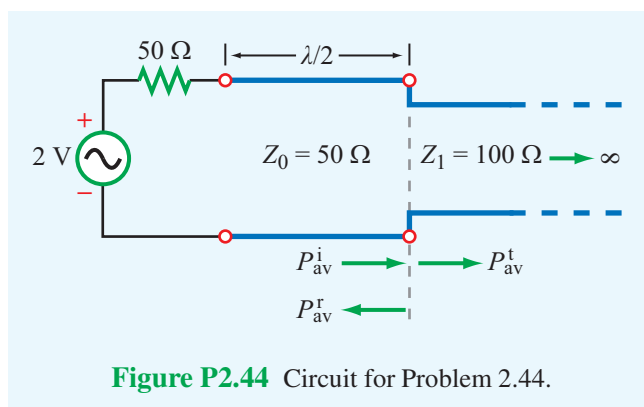


Figure P2.44 Circuit for Problem 2.44.

Solution: Considering the semi-infinite transmission line as equivalent to a load (since all power sent down the line is lost to the rest of the circuit), $Z_L = Z_1 = 100\text{ }\Omega$. Since the feed line is $\lambda/2$ in length, Eq. (2.96) gives $Z_{in} = Z_L = 100\text{ }\Omega$ and $\beta l = (2\pi/\lambda)(\lambda/2) = \pi$, so $\exp \pm j\beta l = -1$. Hence

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = \frac{1}{3}.$$

Also, converting the generator to a phasor gives $\tilde{V}_g = 2 \exp j0^\circ$ (V). Plugging all these results into Eq. (2.82),

$$\begin{aligned} V_0^+ &= \left(\frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}} \right) \left(\frac{1}{\exp j\beta l + \Gamma \exp -j\beta l} \right) = \left(\frac{2 \times 100}{50 + 100} \right) \left[\frac{1}{(-1) + \frac{1}{3}(-1)} \right] \\ &= 1 \exp j180^\circ = -1 \quad (\text{V}). \end{aligned}$$

From Eqs. (2.104), (2.105), and (2.106),

$$\begin{aligned} P_{av}^i &= \frac{|V_0^+|^2}{2Z_0} = \frac{|1 \exp j180^\circ|^2}{2 \times 50} = 10.0 \text{ mW}, \\ P_{av}^r &= -|\Gamma|^2 P_{av}^i = -\left| \frac{1}{3} \right|^2 \times 10 \text{ mW} = -1.1 \text{ mW}, \\ P_{av} &= P_{av}^t = P_{av}^i + P_{av}^r = 10.0 \text{ mW} - 1.1 \text{ mW} = 8.9 \text{ mW}. \end{aligned}$$