

2.46 An antenna with a load impedance

$$Z_L = (75 + j25) \Omega$$

is connected to a transmitter through a $50\text{-}\Omega$ lossless transmission line. If under matched conditions ($50\text{-}\Omega$ load) the transmitter can deliver 20 W to the load, how much power can it deliver to the antenna? Assume that $Z_g = Z_0$.

Solution: From Eqs. (2.82) and (2.76),

$$\begin{aligned} V_0^+ &= \left(\frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}} \right) \left(\frac{1}{\exp j\beta l + \Gamma \exp -j\beta l} \right) \\ &= \left\{ \frac{\tilde{V}_g Z_0 [(1 + \Gamma \exp -j2\beta l)/(1 - \Gamma \exp -j2\beta l)]}{Z_0 + Z_0 [(1 + \Gamma \exp -j2\beta l)/(1 - \Gamma \exp -j2\beta l)]} \right\} \left(\frac{\exp -j\beta l}{1 + \Gamma \exp -j2\beta l} \right) \\ &= \frac{\tilde{V}_g \exp -j\beta l}{(1 - \Gamma \exp -j2\beta l) + (1 + \Gamma \exp -j2\beta l)} \\ &= \frac{\tilde{V}_g \exp -j\beta l}{(1 - \Gamma \exp -j2\beta l) + (1 + \Gamma \exp -j2\beta l)} = \frac{1}{2} \tilde{V}_g \exp -j\beta l. \end{aligned}$$

Thus, in Eq. (2.106),

$$P_{av} = \frac{|V_0^+|^2}{2Z_0} (1 - |\Gamma|^2) = \frac{|\frac{1}{2} \tilde{V}_g \exp -j\beta l|^2}{2Z_0} (1 - |\Gamma|^2) = \frac{|\tilde{V}_g|^2}{8Z_0} (1 - |\Gamma|^2).$$

Under the matched condition, $|\Gamma| = 0$ and $P_L = 20\text{ W}$, so $|\tilde{V}_g|^2/8Z_0 = 20\text{ W}$.

When $Z_L = (75 + j25) \Omega$, from Eq. (2.59a),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(75 + j25) \Omega - 50 \Omega}{(75 + j25) \Omega + 50 \Omega} = 0.277 \exp j33.6^\circ,$$

so $P_{av} = 20\text{ W} (1 - |\Gamma|^2) = 20\text{ W} (1 - 0.277^2) = 18.46\text{ W}$.
