3.43 For the scalar function $U = \frac{1}{R}\sin^2\theta$, determine its directional derivative along the range direction $\hat{\mathbf{R}}$ and then evaluate it at $P = (5, \pi/4, \pi/2)$.

Solution:

$$U = \frac{1}{R}\sin^2\theta,$$

$$\nabla U = \hat{\mathbf{R}}\frac{\partial U}{\partial R} + \hat{\boldsymbol{\theta}}\frac{1}{R}\frac{\partial U}{\partial \theta} + \hat{\boldsymbol{\phi}}\frac{1}{R\sin\theta}\frac{\partial U}{\partial \phi} = -\hat{\mathbf{R}}\frac{\sin^2\theta}{R^2} - \hat{\boldsymbol{\theta}}\frac{2\sin\theta\cos\theta}{R},$$

$$\frac{dU}{dl} = \nabla U \cdot \hat{\mathbf{R}} = -\frac{\sin^2\theta}{R^2},$$

$$\left.\frac{dU}{dl}\right|_{(5,\pi/4,\pi/2)} = -\frac{\sin^2(\pi/4)}{25} = -0.02.$$