3.45 Vector field $\mathbf{E}$ is characterized by the following properties: (a) $\mathbf{E}$ points along $\hat{\mathbf{R}}$, (b) the magnitude of $\mathbf{E}$ is a function of only the distance from the origin, (c) $\mathbf{E}$ vanishes at the origin, and (d) $\nabla \cdot \mathbf{E} = 12$, everywhere. Find an expression for $\mathbf{E}$ that satisfies these properties.

Solution: According to properties (a) and (b), $\mathbf{E}$ must have the form

$$\mathbf{E} = \hat{\mathbf{R}} E_R$$

where E_R is a function of R only.

$$\begin{aligned}
\nabla \cdot \mathbf{E} = \frac{1}{R^2} \frac{\partial}{\partial R}(R^2 E_R) &= 12, \\
\frac{\partial}{\partial R}(R^2 E_R) &= 12R^2, \\
\int_0^R \frac{\partial}{\partial R}(R^2 E_R)\, dR &= \int_0^R 12R^2\, dR, \\
R^2 E_R \Big|_0^R &= \frac{12R^3}{3}\Bigg|_0^R, \\
R^2 E_R &= 4R^3.
\end{aligned}$$

Hence,

$$E_R = 4R,$$

and

$$\mathbf{E} = \hat{\mathbf{R}} 4R.$$