

3.57 Find the Laplacian of the following scalar functions:

- (a) $V = 4xy^2z^3$,
- (b) $V = xy + yz + zx$,
- (c) $V = 3/(x^2 + y^2)$,
- (d) $V = 5e^{-r} \cos \phi$,
- (e) $V = 10e^{-R} \sin \theta$.

Solution:

(a) From Eq. (3.110), $\nabla^2 (4xy^2z^3) = 8xz^3 + 24xy^2z$.

(b) $\nabla^2 (xy + yz + zx) = 0$.

(c) From the inside back cover of the book,

$$\nabla^2 \left(\frac{3}{x^2 + y^2} \right) = \nabla^2 (3r^{-2}) = 12r^{-4} = \frac{12}{(x^2 + y^2)^2}.$$

(d)

$$\nabla^2 (5 \exp -r \cos \phi) = 5 \exp -r \cos \phi \left(1 - \frac{1}{r} - \frac{1}{r^2} \right).$$

(e)

$$\nabla^2 (10 \exp -R \sin \theta) = 10 \exp -R \left[\sin \theta \left(1 - \frac{2}{R} \right) + \frac{\cos^2 \theta - \sin^2 \theta}{R^2 \sin \theta} \right].$$
