3.6 Given vectors $\mathbf{A} = \hat{\mathbf{x}}2 - \hat{\mathbf{y}} + \hat{\mathbf{z}}3$ and $\mathbf{B} = \hat{\mathbf{x}}3 - \hat{\mathbf{z}}2$, find a vector $\mathbf{C}$ whose magnitude is 9 and whose direction is perpendicular to both $\mathbf{A}$ and $\mathbf{B}$.

Solution: The cross product of two vectors produces a new vector which is perpendicular to both of the original vectors. Two vectors exist which have a magnitude of 9 and are orthogonal to both $\vec{A}$ and $\vec{B}$: one which is 9 units long in the direction of the unit vector parallel to $\vec{A} \times \vec{B}$, and one in the opposite direction.

$$\begin{aligned}
\vec{C} = \pm 9 \frac{\vec{A} \times \vec{B}}{\left|\vec{A} \times \vec{B}\right|} &= \pm 9 \frac{(\hat{\mathbf{x}}2 - \hat{\mathbf{y}} + \hat{\mathbf{z}}3) \times (\hat{\mathbf{x}}3 - \hat{\mathbf{z}}2)}{|(\hat{\mathbf{x}}2 - \hat{\mathbf{y}} + \hat{\mathbf{z}}3) \times (\hat{\mathbf{x}}3 - \hat{\mathbf{z}}2)|} \\
&= \pm 9 \frac{\hat{\mathbf{x}}2 + \hat{\mathbf{y}}13 + \hat{\mathbf{z}}3}{\sqrt{2^2 + 13^2 + 3^2}} \approx \pm (\hat{\mathbf{x}}1.34 + \hat{\mathbf{y}}8.67 + \hat{\mathbf{z}}2.0)\,.
\end{aligned}$$