

4.3 Find the total charge contained in a round-top cone defined by $R \leq 2$ m and $0 \leq \theta \leq \pi/4$, given that $\rho_v = 10R^2 \cos^2 \theta$ (mC/m³).

Solution: For the cone of Fig. P4.3, application of Eq. (4.5) gives

$$\begin{aligned}
 Q &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/4} \int_{R=0}^2 10R^2 \cos^2 \theta R^2 \sin \theta dR d\theta d\phi \\
 &= \left(\frac{-2}{3} R^5 \phi \cos^3 \theta \right) \bigg|_{R=0}^2 \bigg|_{\theta=0}^{\pi/4} \bigg|_{\phi=0}^{2\pi} \\
 &= \frac{128\pi}{3} \left(1 - \left(\frac{\sqrt{2}}{2} \right)^3 \right) = 86.65 \text{ (mC)}.
 \end{aligned}$$

