

4.56 Figure P4.56(a) depicts a capacitor consisting of two parallel, conducting plates separated by a distance d . The space between the plates contains two adjacent dielectrics, one with permittivity ϵ_1 and surface area A_1 and another with ϵ_2 and A_2 . The objective of this problem is to show that the capacitance C of the configuration shown in Fig. P4.56(a) is equivalent to two capacitances in parallel, as illustrated in Fig. P4.56(b), with

$$C = C_1 + C_2 \quad (19)$$

where

$$C_1 = \frac{\epsilon_1 A_1}{d} \quad (20)$$

$$C_2 = \frac{\epsilon_2 A_2}{d} \quad (21)$$

To this end, proceed as follows:

- (a) Find the electric fields \mathbf{E}_1 and \mathbf{E}_2 in the two dielectric layers.
- (b) Calculate the energy stored in each section and use the result to calculate C_1 and C_2 .
- (c) Use the total energy stored in the capacitor to obtain an expression for C . Show that (23) is indeed a valid result.

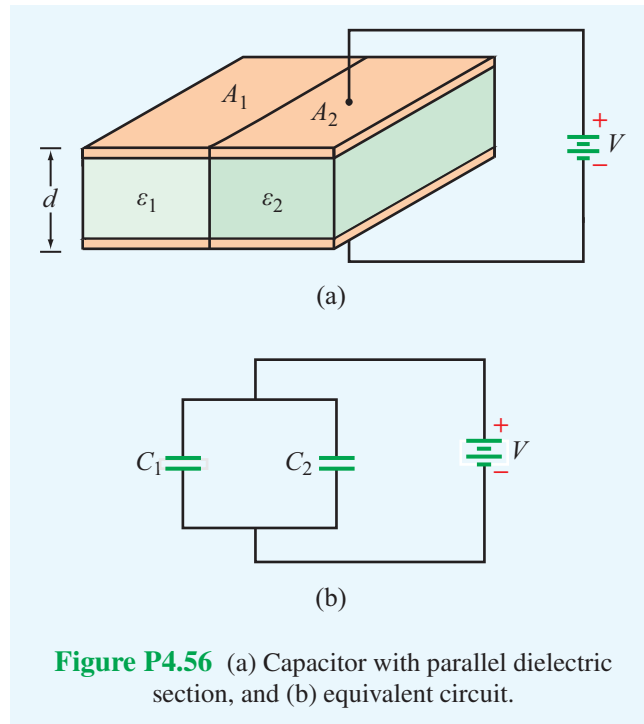
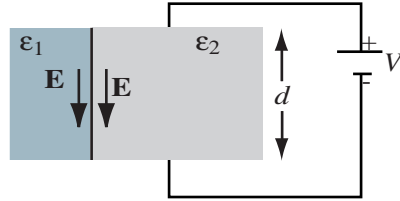


Figure P4.56 (a) Capacitor with parallel dielectric section, and (b) equivalent circuit.

Solution:



(c)

Figure P4.56 (c) Electric field inside of capacitor.

(a) Within each dielectric section, \mathbf{E} will point from the plate with positive voltage to the plate with negative voltage, as shown in Fig. P4-56(c). From $V = Ed$,

$$E_1 = E_2 = \frac{V}{d}.$$

(b)

$$W_{e1} = \frac{1}{2} \epsilon_1 E_1^2 \cdot \mathcal{V} = \frac{1}{2} \epsilon_1 \frac{V^2}{d^2} \cdot A_1 d = \frac{1}{2} \epsilon_1 V^2 \frac{A_1}{d}.$$

But, from Eq. (4.121),

$$W_{e1} = \frac{1}{2} C_1 V^2.$$

Hence $C_1 = \epsilon_1 \frac{A_1}{d}$. Similarly, $C_2 = \epsilon_2 \frac{A_2}{d}$.

(c) Total energy is

$$W_e = W_{e1} + W_{e2} = \frac{1}{2} \frac{V^2}{d} (\epsilon_1 A_1 + \epsilon_2 A_2) = \frac{1}{2} C V^2.$$

Hence,

$$C = \frac{\epsilon_1 A_1}{d} + \frac{\epsilon_2 A_2}{d} = C_1 + C_2.$$
