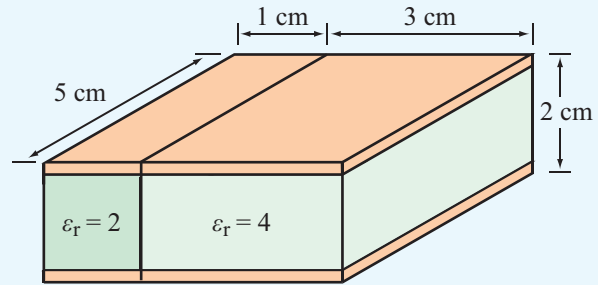
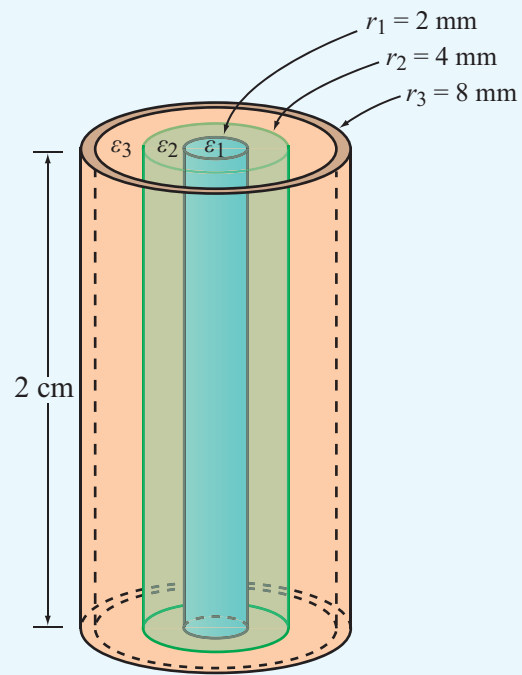


4.57 Use the result of Problem 4.56 to determine the capacitance for each of the following configurations:

- (a) Conducting plates are on top and bottom faces of the rectangular structure in Fig. P4.57(a).
- (b) Conducting plates are on front and back faces of the structure in Fig. P4.57(a).
- (c) Conducting plates are on top and bottom faces of the cylindrical structure in Fig. P4.57(b).



(a)



$$\epsilon_1 = 8\epsilon_0; \epsilon_2 = 4\epsilon_0; \epsilon_3 = 2\epsilon_0$$

(b)

Figure P4.57 Dielectric sections for Problems 4.57 and 4.59.

Solution:

(a) The two capacitors share the same voltage; hence they are in parallel.

$$C_1 = \epsilon_1 \frac{A_1}{d} = 2\epsilon_0 \frac{(5 \times 1) \times 10^{-4}}{2 \times 10^{-2}} = 5\epsilon_0 \times 10^{-2},$$

$$C_2 = \epsilon_2 \frac{A_2}{d} = 4\epsilon_0 \frac{(5 \times 3) \times 10^{-4}}{2 \times 10^{-2}} = 30\epsilon_0 \times 10^{-2},$$

$$C = C_1 + C_2 = (5\epsilon_0 + 30\epsilon_0) \times 10^{-2} = 0.35\epsilon_0 = 3.1 \times 10^{-12} \text{ F}.$$

(b)

$$C_1 = \epsilon_1 \frac{A_1}{d} = 2\epsilon_0 \frac{(2 \times 1) \times 10^{-4}}{5 \times 10^{-2}} = 0.8\epsilon_0 \times 10^{-2},$$

$$C_2 = \epsilon_2 \frac{A_2}{d} = 4\epsilon_0 \frac{(3 \times 2) \times 10^{-4}}{5 \times 10^{-2}} = \frac{24}{5}\epsilon_0 \times 10^{-2},$$

$$C = C_1 + C_2 = 0.5 \times 10^{-12} \text{ F}.$$

(c)

$$C_1 = \epsilon_1 \frac{A_1}{d} = 8\epsilon_0 \frac{(\pi r_1^2)}{2 \times 10^{-2}} = \frac{4\pi\epsilon_0}{10^{-2}} (2 \times 10^{-3})^2 = 0.04 \times 10^{-12} \text{ F},$$

$$\begin{aligned} C_2 &= \epsilon_2 \frac{A_2}{d} \\ &= 4\epsilon_0 \frac{(\pi(r_2^2 - r_1^2))}{2 \times 10^{-2}} = \frac{2\pi\epsilon_0}{10^{-2}} [(4 \times 10^{-3})^2 - (2 \times 10^{-3})^2] = 0.06 \times 10^{-12} \text{ F}, \end{aligned}$$

$$\begin{aligned} C_3 &= \epsilon_3 \frac{A_3}{d} \\ &= 2\epsilon_0 \frac{(\pi(r_3^2 - r_2^2))}{2 \times 10^{-2}} = \frac{\pi\epsilon_0}{10^{-2}} [(8 \times 10^{-3})^2 - (4 \times 10^{-3})^2] = 0.12 \times 10^{-12} \text{ F}, \end{aligned}$$

$$C = C_1 + C_2 + C_3 = 0.22 \times 10^{-12} \text{ F}.$$
