

6.28 In free space, the magnetic field is given by

$$\mathbf{H} = \hat{\phi} \frac{36}{r} \cos(6 \times 10^9 t - kz) \quad (\text{mA/m}).$$

- (a) Determine k .
- (b) Determine \mathbf{E} .
- (c) Determine \mathbf{J}_d .

Solution:

(a) From the given expression, $\omega = 6 \times 10^9$ (rad/s), and since the medium is free space,

$$k = \frac{\omega}{c} = \frac{6 \times 10^9}{3 \times 10^8} = 20 \quad (\text{rad/m}).$$

(b) Convert \mathbf{H} to phasor:

$$\begin{aligned} \tilde{\mathbf{H}} &= \hat{\phi} \frac{36}{r} e^{-jkz} \quad (\text{mA/m}) \\ \tilde{\mathbf{E}} &= \frac{1}{j\omega\epsilon_0} \nabla \times \tilde{\mathbf{H}} \\ &= \frac{1}{j\omega\epsilon_0} \left[-\hat{\mathbf{r}} \frac{\partial H_\phi}{\partial z} + \hat{\mathbf{z}} \frac{1}{r} \frac{\partial}{\partial r} (r H_\phi) \right] \\ &= \frac{1}{j\omega\epsilon_0} \left[-\hat{\mathbf{r}} \frac{\partial}{\partial z} \left(\frac{36}{r} e^{-jkz} \right) + \hat{\mathbf{z}} \frac{\partial}{\partial r} (36 e^{-jkz}) \right] \\ &= \frac{1}{j\omega\epsilon_0} \left[\hat{\mathbf{r}} \frac{j36k}{r} e^{-jkz} \right] \\ &= \hat{\mathbf{r}} \frac{36k}{\omega\epsilon_0 r} e^{-jkz} = \hat{\mathbf{r}} \frac{36 \times 377}{r} e^{-jkz} \times 10^{-3} = \hat{\mathbf{r}} \frac{13.6}{r} e^{-j20z} \quad (\text{V/m}). \\ \mathbf{E} &= \Re[\tilde{\mathbf{E}} e^{j\omega t}] \\ &= \hat{\mathbf{r}} \frac{13.6}{r} \cos(6 \times 10^9 t - 20z) \quad (\text{V/m}). \end{aligned}$$

(c)

$$\begin{aligned} \mathbf{J}_d &= \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \\ &= \hat{\mathbf{r}} \frac{13.6}{r} \epsilon_0 \frac{\partial}{\partial t} (\cos(6 \times 10^9 t - 20z)) \\ &= -\hat{\mathbf{r}} \frac{13.6\epsilon_0 \times 6 \times 10^9}{r} \sin(6 \times 10^9 t - 20z) \quad (\text{A/m}^2) \end{aligned}$$

$$= -\hat{\mathbf{r}} \frac{0.72}{r} \sin(6 \times 10^9 t - 20z) \quad (\text{A/m}^2).$$
