

**6.29** The magnetic field in a given dielectric medium is given by

$$\mathbf{H} = \hat{\mathbf{y}} 6 \cos 2z \sin(2 \times 10^7 t - 0.1x) \quad (\text{A/m}),$$

where  $x$  and  $z$  are in meters. Determine:

- (a)  $\mathbf{E}$ ,
- (b) the displacement current density  $\mathbf{J}_d$ , and
- (c) the charge density  $\rho_v$ .

**Solution:**

(a)

$$\mathbf{H} = \hat{\mathbf{y}} 6 \cos 2z \sin(2 \times 10^7 t - 0.1x) = \hat{\mathbf{y}} 6 \cos 2z \cos(2 \times 10^7 t - 0.1x - \pi/2),$$

$$\tilde{\mathbf{H}} = \hat{\mathbf{y}} 6 \cos 2z e^{-j0.1x} e^{-j\pi/2} = -\hat{\mathbf{y}} j6 \cos 2z e^{-j0.1x},$$

$$\begin{aligned} \tilde{\mathbf{E}} &= \frac{1}{j\omega\epsilon} \nabla \times \tilde{\mathbf{H}} \\ &= \frac{1}{j\omega\epsilon} \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & -j6 \cos 2z e^{-j0.1x} & 0 \end{vmatrix} \\ &= \frac{1}{j\omega\epsilon} \left\{ \hat{\mathbf{x}} \left[ -\frac{\partial}{\partial z} (-j6 \cos 2z e^{-j0.1x}) \right] + \hat{\mathbf{z}} \left[ \frac{\partial}{\partial x} (-j6 \cos 2z e^{-j0.1x}) \right] \right\} \\ &= \hat{\mathbf{x}} \left( -\frac{12}{\omega\epsilon} \sin 2z e^{-j0.1x} \right) + \hat{\mathbf{z}} \left( \frac{j0.6}{\omega\epsilon} \cos 2z e^{-j0.1x} \right). \end{aligned}$$

From the given expression for  $\mathbf{H}$ ,

$$\omega = 2 \times 10^7 \quad (\text{rad/s}),$$

$$\beta = 0.1 \quad (\text{rad/m}).$$

Hence,

$$u_p = \frac{\omega}{\beta} = 2 \times 10^8 \quad (\text{m/s}),$$

and

$$\epsilon_r = \left( \frac{c}{u_p} \right)^2 = \left( \frac{3 \times 10^8}{2 \times 10^8} \right)^2 = 2.25.$$

Using the values for  $\omega$  and  $\epsilon$ , we have

$$\tilde{\mathbf{E}} = (-\hat{\mathbf{x}} 30 \sin 2z + \hat{\mathbf{z}} j1.5 \cos 2z) \times 10^3 e^{-j0.1x} \quad (\text{V/m}),$$

$$\mathbf{E} = [-\hat{\mathbf{x}} 30 \sin 2z \cos(2 \times 10^7 t - 0.1x) - \hat{\mathbf{z}} 1.5 \cos 2z \sin(2 \times 10^7 t - 0.1x)] \quad (\text{kV/m}).$$

(b)

$$\tilde{\mathbf{D}} = \epsilon \tilde{\mathbf{E}} = \epsilon_r \epsilon_0 \tilde{\mathbf{E}} = (-\hat{\mathbf{x}} 0.6 \sin 2z + \hat{\mathbf{z}} j 0.03 \cos 2z) \times 10^{-6} e^{-j 0.1 x} \quad (\text{C/m}^2),$$

$$\mathbf{J}_d = \frac{\partial \mathbf{D}}{\partial t},$$

or

$$\tilde{\mathbf{J}}_d = j \omega \tilde{\mathbf{D}} = (-\hat{\mathbf{x}} j 12 \sin 2z - \hat{\mathbf{z}} 0.6 \cos 2z) e^{-j 0.1 x},$$

$$\begin{aligned} \mathbf{J}_d &= \Re[\tilde{\mathbf{J}}_d e^{j \omega t}] \\ &= [\hat{\mathbf{x}} 12 \sin 2z \sin(2 \times 10^7 t - 0.1 x) - \hat{\mathbf{z}} 0.6 \cos 2z \cos(2 \times 10^7 t - 0.1 x)] \quad (\text{A/m}^2). \end{aligned}$$

(c) We can find  $\rho_v$  from

$$\nabla \cdot \mathbf{D} = \rho_v$$

or from

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t}.$$

Applying Maxwell's equation,

$$\rho_v = \nabla \cdot \mathbf{D} = \epsilon \nabla \cdot \mathbf{E} = \epsilon_r \epsilon_0 \left( \frac{\partial E_x}{\partial x} + \frac{\partial E_z}{\partial z} \right)$$

yields

$$\begin{aligned} \rho_v &= \epsilon_r \epsilon_0 \left\{ \frac{\partial}{\partial x} [-30 \sin 2z \cos(2 \times 10^7 t - 0.1 x)] \right. \\ &\quad \left. + \frac{\partial}{\partial z} [-1.5 \cos 2z \sin(2 \times 10^7 t - 0.1 x)] \right\} \\ &= \epsilon_r \epsilon_0 [-3 \sin 2z \sin(2 \times 10^7 t - 0.1 x) + 3 \sin 2z \sin(2 \times 10^7 t - 0.1 x)] = 0. \end{aligned}$$