7.28 In a nonmagnetic, lossy, dielectric medium, a 300-MHz plane wave is characterized by the magnetic field phasor

$$\widetilde{\mathbf{H}} = (\hat{\mathbf{x}} - j4\hat{\mathbf{z}})e^{-2y}e^{-j9y} \qquad \text{(A/m)}$$

Obtain time-domain expressions for the electric and magnetic field vectors.

Solution:

$$\widetilde{\mathbf{E}} = -\eta_c \hat{\mathbf{k}} \times \widetilde{\mathbf{H}}.$$

To find η_c, we need ε' and ε''. From the given expression for $\widetilde{\mathbf{H}}$,

$$\alpha = 2 \qquad \text{(Np/m)},$$
$$\beta = 9 \qquad \text{(rad/m)}.$$

Also, we are given than $f = 300$ MHz $= 3 \times 10^8$ Hz. From (7.65a),

$$\alpha^2 - \beta^2 = -\omega^2 \mu \varepsilon',$$
$$4 - 81 = -(2\pi \times 3 \times 10^8)^2 \times 4\pi \times 10^{-7} \times \varepsilon_r' \times \frac{10^{-9}}{36\pi},$$

whose solution gives

$$\varepsilon_r' = 1.95.$$

Similarly, from Eq. (7.65b),

$$2\alpha\beta = \omega^2 \mu \varepsilon'',$$
$$2 \times 2 \times 9 = (2\pi \times 3 \times 10^8)^2 \times 4\pi \times 10^{-7} \times \varepsilon_r'' \times \frac{10^{-9}}{36\pi},$$

which gives

$$\varepsilon_r'' = 0.91.$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon'}} \left(1 - j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$$
$$= \frac{\eta_0}{\sqrt{\varepsilon_r'}} \left(1 - j\frac{0.91}{1.95}\right)^{-1/2} = \frac{377}{\sqrt{1.95}}(0.93 + j0.21) = 256.9\, e^{j12.6^\circ}.$$

Hence,

$$\widetilde{\mathbf{E}} = -256.9\, e^{j12.6^\circ}\, \hat{\mathbf{y}} \times (\hat{\mathbf{x}} - j4\hat{\mathbf{z}})e^{-2y}e^{-j9y}$$
$$= (\hat{\mathbf{x}}\, j4 + \hat{\mathbf{z}})\, 256.9\, e^{-2y} e^{-j9y} e^{j12.6^\circ}$$

$$
\begin{aligned}
&= (\hat{\mathbf{x}}\,4e^{j\pi/2} + \hat{\mathbf{z}})\,256.9\,e^{-2y}e^{-j9y}e^{j12.6^\circ}, \\
\mathbf{E} &= \mathfrak{Re}\{\widetilde{\mathbf{E}}e^{j\omega t}\} \\
&= \hat{\mathbf{x}}\,1.03\times 10^3 e^{-2y}\cos(\omega t - 9y + 102.6^\circ) \\
&\quad + \hat{\mathbf{z}}\,256.9\,e^{-2y}\cos(\omega t - 9y + 12.6^\circ) \qquad \text{(V/m)}, \\
\mathbf{H} &= \mathfrak{Re}\{\widetilde{\mathbf{H}}e^{j\omega t}\} \\
&= \mathfrak{Re}\{(\hat{\mathbf{x}} + j4\hat{\mathbf{z}})e^{-2y}e^{-j9y}e^{j\omega t}\} \\
&= \hat{\mathbf{x}}\,e^{-2y}\cos(\omega t - 9y) + \hat{\mathbf{z}}\,4e^{-2y}\sin(\omega t - 9y) \qquad \text{(A/m)}.
\end{aligned}
$$