7.7 The magnetic field of a plane wave propagating in a nonmagnetic material is given by

$$\mathbf{H} = \hat{\mathbf{x}}\,60\cos(2\pi \times 10^7 t + 0.1\pi y) \\ \hat{\mathbf{z}}\,30\cos(2\pi \times 10^7 t + 0.1\pi y) \qquad (\text{mA/m}).$$

Determine

(a) The wavelength.

(b) ε_r.

(c) **E**.

Solution:

(a) From the expression for **H**, we deduce that

$$\omega = 2\pi \times 10^7 \text{ rad/s} \quad \Rightarrow \quad f = 10^7 \text{ Hz},$$

$$k = 0.1\pi \text{ rad/m} \quad \Rightarrow \quad \lambda = \frac{2\pi}{k} = 20 \text{ m}.$$

(b)

$$u_p = f\lambda = 10^7 \times 20 = 2 \times 10^8 \text{ m/s},$$

$$\varepsilon_r = \left(\frac{c}{u_p}\right)^2 = \left(\frac{3 \times 10^8}{2 \times 10^8}\right)^2 = (1.5)^2 = 2.25.$$

(c) From Eq. (7.39b),

$$\mathbf{E} = -\eta\hat{\mathbf{k}} \times \mathbf{H}.$$

Here,

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{377}{\sqrt{2.25}} = 251.3\ \Omega,$$

$$\hat{\mathbf{k}} = -\hat{\mathbf{y}}.$$

Hence,

$$\mathbf{E} = -251.3\hat{\mathbf{y}} \times [\hat{\mathbf{x}}60\cos(2\pi \times 10^7 t + 0.1\pi y) + \hat{\mathbf{z}}30\cos(2\pi \times 10^7 t + 0.1\pi y)] \times 10^{-3}$$

$$= \hat{\mathbf{z}}15.08\cos(2\pi \times 10^7 t + 0.1\pi y) - \hat{\mathbf{x}}7.54\cos(2\pi \times 10^7 t + 0.1\pi y) \qquad (\text{V/m}).$$