7.8 A 60-MHz plane wave traveling in the $-x$-direction in dry soil with relative permittivity $\varepsilon_r = 4$ has an electric field polarized along the z-direction. Assuming dry soil to be approximately lossless, and given that the magnetic field has a peak value of 10 (mA/m) and that its value was measured to be 7 (mA/m) at $t = 0$ and $x = -0.75$ m, develop complete expressions for the wave's electric and magnetic fields.

Solution: For $f = 60$ MHz $= 6\times10^7$ Hz, $\varepsilon_r = 4$, $\mu_r = 1$,

$$k = \frac{\omega}{c}\sqrt{\varepsilon_r} = \frac{2\pi\times6\times10^7}{3\times10^8}\sqrt{4} = 0.8\pi \qquad \text{(rad/m)}.$$

Given that $\mathbf{E}$ points along $\hat{\mathbf{z}}$ and wave travel is along $-\hat{\mathbf{x}}$, we can write

$$\mathbf{E}(x,t) = \hat{\mathbf{z}}E_0\cos(2\pi\times60\times10^6 t + 0.8\pi x + \phi_0) \qquad \text{(V/m)}$$

where E_0 and ϕ_0 are unknown constants at this time. The intrinsic impedance of the medium is

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{2} = 60\pi \qquad (\Omega).$$

With $\mathbf{E}$ along $\hat{\mathbf{z}}$ and $\hat{\mathbf{k}}$ along $-\hat{\mathbf{x}}$, (7.39) gives

$$\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{k}}\times\mathbf{E}$$

or

$$\mathbf{H}(x,t) = \hat{\mathbf{y}}\frac{E_0}{\eta}\cos(1.2\pi\times10^8 t + 0.8\pi x + \phi_0) \qquad \text{(A/m)}.$$

Hence,

$$\frac{E_0}{\eta} = 10 \qquad \text{(mA/m)}$$

$$E_0 = 10\times60\pi\times10^{-3} = 0.6\pi \qquad \text{(V/m)}.$$

Also,

$$H(-0.75\text{ m},0) = 7\times10^{-3} = 10\cos(-0.8\pi\times0.75 + \phi_0)\times10^{-3}$$

which leads to $\phi_0 = 153.6^\circ$.

Hence,

$$\mathbf{E}(x,t) = \hat{\mathbf{z}}\,0.6\pi\cos(1.2\pi\times10^8 t + 0.8\pi x + 153.6^\circ) \qquad \text{(V/m)}.$$

$$\mathbf{H}(x,t) = \hat{\mathbf{y}}\,10\cos(1.2\pi\times10^8 t + 0.8\pi x + 153.6^\circ) \qquad \text{(mA/m)}.$$