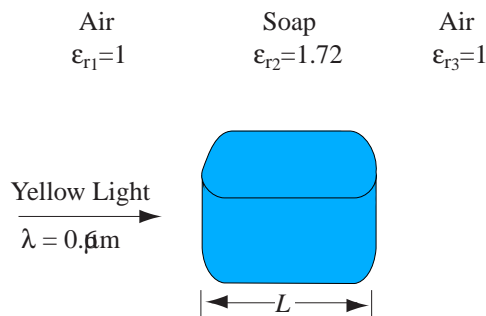


**8.14** Consider a thin film of soap in air under illumination by yellow light with  $\lambda = 0.6 \mu\text{m}$  in vacuum. If the film is treated as a planar dielectric slab with  $\epsilon_r = 1.72$ , surrounded on both sides by air, what film thickness would produce strong reflection of the yellow light at normal incidence?

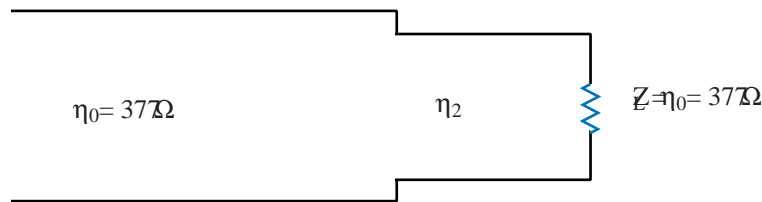
**Solution:** The transmission line analogue of the soap-bubble wave problem is shown in Fig. P8.14(b) where the load  $Z_L$  is equal to  $\eta_0$ , the impedance of the air medium on the other side of the bubble. That is,

$$\eta_0 = 377 \Omega, \quad \eta_1 = \frac{377}{\sqrt{1.72}} = 287.5 \Omega.$$

The normalized load impedance is



(a) Yellow light incident on soap bubble.



(b) Transmission-line equivalent circuit

**Figure P8.14** Diagrams for Problem 8.14.

$$z_L = \frac{\eta_0}{\eta_1} = 1.31.$$

For the reflection by the soap bubble to be the largest,  $Z_{in}$  needs to be the most

different from  $\eta_0$ . This happens when  $z_L$  is transformed through a length  $\lambda/4$ . Hence,

$$L = \frac{\lambda}{4} = \frac{\lambda_0}{4\sqrt{\epsilon_r}} = \frac{0.6 \mu\text{m}}{4\sqrt{1.72}} = 0.115 \mu\text{m},$$

where  $\lambda$  is the wavelength of the soap bubble material. Strong reflections will also occur if the thickness is greater than  $L$  by integer multiples of  $n\lambda/2 = (0.23n) \mu\text{m}$ .

Hence, in general

$$L = (0.115 + 0.23n) \mu\text{m}, \quad n = 0, 1, 2, \dots$$

According to Section 2-7.5, transforming a load  $Z_L = 377 \Omega$  through a  $\lambda/4$  section of  $Z_0 = 287.5 \Omega$  ends up presenting an input impedance of

$$Z_{\text{in}} = \frac{Z_0^2}{Z_L} = \frac{(287.5)^2}{377} = 219.25 \Omega.$$

This  $Z_{\text{in}}$  is at the input side of the soap bubble. The reflection coefficient at that interface is

$$\Gamma = \frac{Z_{\text{in}} - \eta_0}{Z_{\text{in}} + \eta_0} = \frac{219.25 - 377}{219.25 + 377} = -0.27.$$

Any other thickness would produce a reflection coefficient with a smaller magnitude.

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