

8.17 A light ray is incident on a prism in air at an angle θ as shown in Fig. P8.17. The ray is refracted at the first surface and again at the second surface. In terms of the apex angle ϕ of the prism and its index of refraction n , determine the smallest value of θ for which the ray will emerge from the other side. Find this minimum θ for $n = 1.4$ and $\phi = 60^\circ$.

Solution:

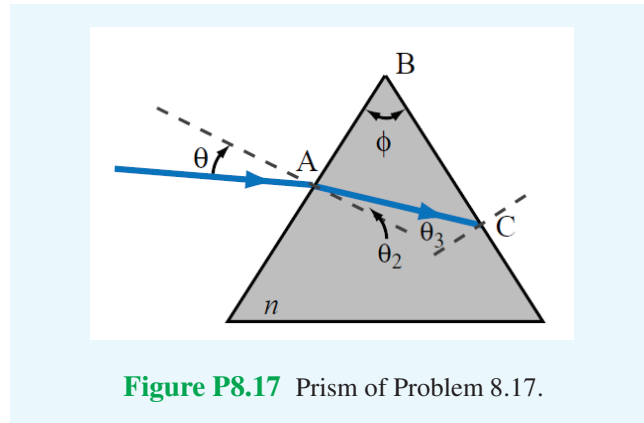


Figure P8.17 Prism of Problem 8.17.

For the beam to emerge at the second boundary, it is necessary that

$$\theta_3 < \theta_c,$$

where $\sin \theta_c = 1/n$. From the geometry of triangle ABC ,

$$180^\circ = \phi + (90^\circ - \theta_2) + (90^\circ - \theta_3),$$

or $\theta_2 = \phi - \theta_3$. At the first boundary, $\sin \theta = n \sin \theta_2$. Hence,

$$\sin \theta_{\min} = n \sin(\phi - \theta_3) = n \sin \left(\phi - \sin^{-1} \left(\frac{1}{n} \right) \right),$$

or

$$\theta_{\min} = \sin^{-1} \left[n \sin \left(\phi - \sin^{-1} \left(\frac{1}{n} \right) \right) \right].$$

For $n = 1.4$ and $\phi = 60^\circ$,

$$\theta_{\min} = \sin^{-1} \left[1.4 \sin \left(60^\circ - \sin^{-1} \left(\frac{1}{1.4} \right) \right) \right] = 20.4^\circ.$$