

8.8 Repeat Problem 8.6, but replace the dielectric medium with a conductor with $\epsilon_r = 1$, $\mu_r = 1$, and $\sigma = 2.78 \times 10^{-3}$ S/m.

Solution:

(a) Medium 1:

$$\eta_1 = \eta_0 = 120\pi = 377 \quad (\Omega), \quad \lambda_1 = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^7} = 6 \text{ m},$$

Medium 2:

$$\frac{\sigma_2}{\omega \epsilon_2} = \frac{2.78 \times 10^{-3} \times 36\pi}{2\pi \times 5 \times 10^7 \times 10^{-9}} = 1.$$

Hence, Medium 2 is a quasi-conductor. From Eq. (7.70),

$$\begin{aligned} \eta_2 &= \sqrt{\frac{\mu_2}{\epsilon_2}} \left(1 - j \frac{\epsilon_2''}{\epsilon_2'} \right)^{-1/2} = 120\pi \left(1 - j \frac{\sigma_2}{\omega \epsilon_2} \right)^{-1/2} \\ &= 120\pi (1 - j1)^{-1/2} \\ &= 120\pi (\sqrt{2})^{-1/2} e^{j22.5^\circ} = (292.88 + j121.31) \quad (\Omega). \end{aligned}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{(292.88 + j121.31) - 377}{(292.88 + j121.31) + 377} = -0.09 + j0.12 = 0.22 \angle 114.5^\circ.$$

(b)

$$\begin{aligned} S_{\text{av}}^i &= \frac{|E_0^i|^2}{2\eta_1} = \frac{50^2}{2 \times 120\pi} = 3.32 \quad (\text{W/m}^2), \\ |S_{\text{av}}^r| &= |\Gamma|^2 S_{\text{av}}^i = (0.22)^2 (3.32) = 0.16 \quad (\text{W/m}^2). \end{aligned}$$

(c) In medium 1 (air),

$$\lambda_1 = \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^7} = 6 \text{ m}.$$

For $\theta_r = 114.5^\circ = 2 \text{ rad}$, Eqs. (8.16) and (8.17) give

$$\begin{aligned} l_{\text{max}} &= \frac{\theta_r \lambda_1}{4\pi} + \frac{(0)\lambda_1}{2} = \frac{2(6)}{4} + 0 = 3 \text{ m}, \\ l_{\text{min}} &= l_{\text{max}} - \frac{\lambda_1}{4} = 3 - \frac{6}{4} = 3 - 1.5 = 1.5 \text{ m}. \end{aligned}$$
