

9.37 A two-element array consisting of two isotropic antennas separated by a distance d along the z axis is placed in a coordinate system whose z axis points eastward and whose x axis points toward the zenith. If a_0 and a_1 are the amplitudes of the excitations of the antennas at $z = 0$ and at $z = d$, respectively, and if δ is the phase of the excitation of the antenna at $z = d$ relative to that of the other antenna, find the array factor and plot the pattern in the x - z plane for the following:

- (a) $a_0 = a_1 = 1$, $\delta = \pi/4$, and $d = \lambda/2$
- (b) $a_0 = 1$, $a_1 = 2$, $\delta = 0$, and $d = \lambda$
- (c) $a_0 = a_1 = 1$, $\delta = -\pi/2$, and $d = \lambda/2$
- (d) $a_0 = 1$, $a_1 = 2$, $\delta = \pi/4$, and $d = \lambda/2$
- (e) $a_0 = 1$, $a_1 = 2$, $\delta = \pi/2$, and $d = \lambda/4$

Solution:

(e) Employing Eq. (9.110), and setting $a_0 = 1$, $a_1 = 2$, $\psi_0 = 0$, $\psi_1 = \delta = \pi/2$, and $d = \lambda/4$, we have

$$\begin{aligned}
 F_a(\theta) &= \left| \sum_{i=0}^1 a_i e^{j\psi_i} e^{j i k d \cos \theta} \right|^2 \\
 &= \left| 1 + 2e^{j\pi/2} e^{j(2\pi/\lambda)(\lambda/4) \cos \theta} \right|^2 \\
 &= \left| 1 + 2e^{j(\pi \cos \theta + \pi)/2} \right|^2 \\
 &= 5 + 4 \cos \left(\frac{\pi}{2} \cos \theta + \frac{\pi}{2} \right) = 5 - 4 \sin \left(\frac{\pi}{2} \cos \theta \right).
 \end{aligned}$$

A plot of the array factor is shown in Fig. 9-37(e).

