9.44 A five-element equally spaced linear array with $d = \lambda/2$ is excited with uniform phase and an amplitude distribution given by the binomial distribution

$$a_i = \frac{(N-1)!}{i!(N-i-1)!}, \qquad i = 0, 1, \ldots, (N-1),$$

where N is the number of elements. Develop an expression for the array factor.

Solution: Using the given formula,

$$a_0 = \frac{(5-1)!}{0!4!} = 1 \qquad \text{(note that } 0! = 1\text{)}$$

$$a_1 = \frac{4!}{1!3!} = 4$$

$$a_2 = \frac{4!}{2!2!} = 6$$

$$a_3 = \frac{4!}{3!1!} = 4$$

$$a_4 = \frac{4!}{0!4!} = 1$$

Application of (9.113) leads to:

$$\begin{aligned}
F_a(\gamma) &= \left| \sum_{i=0}^{N-1} a_i e^{ji\gamma} \right|^2, \qquad \gamma = \frac{2\pi d}{\lambda} \cos\theta \\
&= \left| 1 + 4e^{j\gamma} + 6e^{j2\gamma} + 4e^{j3\gamma} + e^{j4\gamma} \right|^2 \\
&= \left| e^{j2\gamma}(e^{-j2\gamma} + 4e^{-j\gamma} + 6 + 4e^{j\gamma} + e^{j2\gamma}) \right|^2 \\
&= (6 + 8\cos\gamma + 2\cos 2\gamma)^2.
\end{aligned}$$

With $d = \lambda/2$, $\gamma = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos\theta = \pi\cos\theta$,

$$F_a(\theta) = [6 + 8\cos(\pi\cos\theta) + 2\cos(2\pi\cos\theta)]^2.$$