**1.15** A laser beam traveling through fog was observed to have an intensity of 1 ($\mu$W/m$^2$) at a distance of 2 m from the laser gun and an intensity of 0.2 ($\mu$W/m$^2$) at a distance of 3 m. Given that the intensity of an electromagnetic wave is proportional to the square of its electric-field amplitude, find the attenuation constant $\alpha$ of fog.

**Solution:** If the electric field is of the form

$$E(x,t) = E_0 e^{-\alpha x}\cos(\omega t - \beta x),$$

then the intensity must have a form

$$\begin{aligned} I(x,t) &\approx [E_0 e^{-\alpha x}\cos(\omega t - \beta x)]^2 \\ &\approx E_0^2 e^{-2\alpha x}\cos^2(\omega t - \beta x) \end{aligned}$$

or

$$I(x,t) = I_0 e^{-2\alpha x}\cos^2(\omega t - \beta x)$$

where we define $I_0 \approx E_0^2$. We observe that the magnitude of the intensity varies as $I_0 e^{-2\alpha x}$. Hence,

$$\begin{aligned} &\text{at } x = 2 \text{ m}, \qquad I_0 e^{-4\alpha} = 1 \times 10^{-6} \qquad (\text{W/m}^2), \\ &\text{at } x = 3 \text{ m}, \qquad I_0 e^{-6\alpha} = 0.2 \times 10^{-6} \qquad (\text{W/m}^2). \end{aligned}$$

$$\begin{aligned} \frac{I_0 e^{-4\alpha}}{I_0 e^{-6\alpha}} &= \frac{10^{-6}}{0.2 \times 10^{-6}} = 5 \\ e^{-4\alpha} \cdot e^{6\alpha} &= e^{2\alpha} = 5 \\ \alpha &= 0.8 \qquad (\text{NP/m}). \end{aligned}$$