

1.25 A voltage source given by

$$v_s(t) = 25 \cos(2\pi \times 10^3 t - 30^\circ) \quad (\text{V})$$

is connected to a series RC load as shown in Fig. 1-20. If $R = 1 \text{ M}\Omega$ and $C = 200 \text{ pF}$, obtain an expression for $v_c(t)$, the voltage across the capacitor.

Solution: In the phasor domain, the circuit is a voltage divider, and

$$\tilde{V}_c = \tilde{V}_s \frac{1/j\omega C}{R + 1/j\omega C} = \frac{\tilde{V}_s}{(1 + j\omega RC)}.$$

Now $\tilde{V}_s = 25 \exp -j30^\circ \text{ V}$ with $\omega = 2\pi \times 10^3 \text{ rad/s}$, so

$$\begin{aligned} \tilde{V}_c &= \frac{25 \exp -j30^\circ \text{ V}}{1 + j((2\pi \times 10^3 \text{ rad/s}) \times (10^6 \Omega) \times (200 \times 10^{-12} \text{ F}))} \\ &= \frac{25 \exp -j30^\circ \text{ V}}{1 + j2\pi/5} = 15.57 \exp -j81.5^\circ \text{ V}. \end{aligned}$$

Converting back to an instantaneous value,

$$v_c(t) = \Re \tilde{V}_c \exp j\omega t = \Re 15.57 \exp j(\omega t - 81.5^\circ) \text{ V} = 15.57 \cos(2\pi \times 10^3 t - 81.5^\circ) \text{ V},$$

where t is expressed in seconds.
