1.9 Give expressions for $y(x,t)$ for a sinusoidal wave traveling along a string in the negative x-direction, given that $y_{\max} = 40$ cm, $\lambda = 30$ cm, $f = 10$ Hz, and

(a) $y(x,0) = 0$ at $x = 0$,

(b) $y(x,0) = 0$ at $x = 3.75$ cm.

Solution: For a wave traveling in the negative x-direction, we use Eq. (1.17) with $\omega = 2\pi f = 20\pi$ (rad/s), $\beta = 2\pi/\lambda = 2\pi/0.3 = 20\pi/3$ (rad/s), $A = 40$ cm, and x assigned a positive sign:

$$y(x,t) = 40\cos\left(20\pi t + \frac{20\pi}{3}x + \phi_0\right) \qquad \text{(cm)},$$

with x in meters.

(a) $y(0,0) = 0 = 40\cos\phi_0$. Hence, $\phi_0 = \pm\pi/2$, and

$$\begin{aligned} y(x,t) &= 40\cos\left(20\pi t + \frac{20\pi}{3}x \pm \frac{\pi}{2}\right) \\ &= \begin{cases} -40\sin\left(20\pi t + \frac{20\pi}{3}x\right) \text{ (cm), if } \phi_0 = \pi/2, \\ 40\sin\left(20\pi t + \frac{20\pi}{3}x\right) \text{ (cm), if } \phi_0 = -\pi/2. \end{cases} \end{aligned}$$

(b) At $x = 3.75$ cm $= 3.75 \times 10^{-2}$ m, $y = 0 = 40\cos(\pi/4 + \phi_0)$. Hence, $\phi_0 = \pi/4$ or $5\pi/4$, and

$$y(x,t) = \begin{cases} 40\cos\left(20\pi t + \frac{20\pi}{3}x + \frac{\pi}{4}\right) \text{ (cm), if } \phi_0 = \pi/4, \\ 40\cos\left(20\pi t + \frac{20\pi}{3}x + \frac{5\pi}{4}\right) \text{ (cm), if } \phi_0 = 5\pi/4. \end{cases}$$