

2.19 A $50\text{-}\Omega$ lossless transmission line is terminated in a load with impedance $Z_L = (30 - j50)\text{ }\Omega$. The wavelength is 8 cm. Find:

- (a) the reflection coefficient at the load,
- (b) the standing-wave ratio on the line,
- (c) the position of the voltage maximum nearest the load,
- (d) the position of the current maximum nearest the load.
- (e) Verify quantities in parts (a)–(d) using CD Module 2.4. Include a printout of the screen display.

Solution:

- (a) From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 - j50) - 50}{(30 - j50) + 50} = 0.57 \exp -j79.8^\circ.$$

- (b) From Eq. (2.73),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.57}{1 - 0.57} = 3.65.$$

- (c) From Eq. (2.70)

$$\begin{aligned} d_{\max} &= \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} = \frac{-79.8^\circ \times 8\text{ cm}}{4\pi} \frac{\pi\text{ rad}}{180^\circ} + \frac{n \times 8\text{ cm}}{2} \\ &= -0.89\text{ cm} + 4.0\text{ cm} = 3.11\text{ cm}. \end{aligned}$$

- (d) A current maximum occurs at a voltage minimum, and from Eq. (2.72),

$$d_{\min} = d_{\max} - \lambda/4 = 3.11\text{ cm} - 8\text{ cm}/4 = 1.11\text{ cm}.$$

- (e) The problem statement does not specify the frequency, so in Module 2.4 we need to select the combination of f and ϵ_r such that $\lambda = 5\text{ cm}$. With ϵ_r chosen as 1,

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{8 \times 10^{-2}} = 3.75\text{ GHz}.$$

The generator parameters are irrelevant to the problem.

The results listed in the output screens are very close to those given in parts (a) through (d).

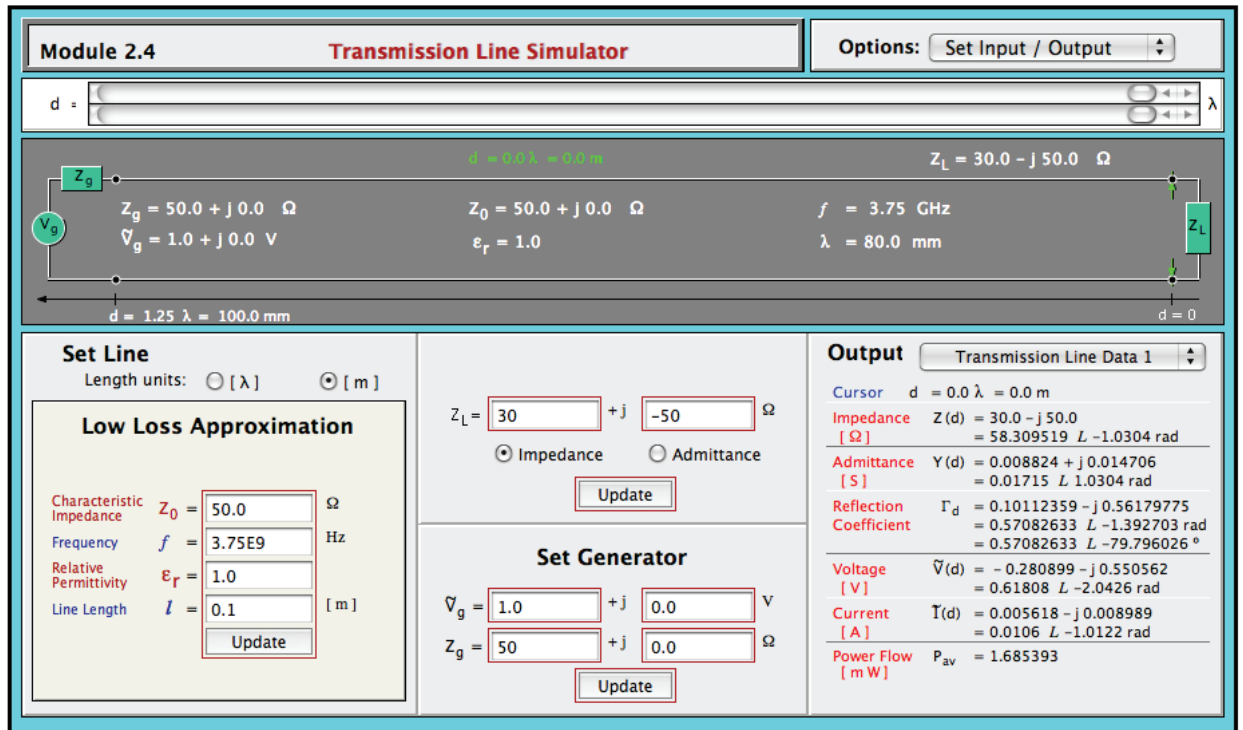


Figure P2.19(a)

Module 2.4
Transmission Line Simulator
Options: Set Input / Output

d =
λ

$Z_g = 50.0 + j\,0.0\ \Omega$
 $V_g = 1.0 + j\,0.0\ \text{V}$
 $Z_0 = 50.0 + j\,0.0\ \Omega$
 $\epsilon_r = 1.0$
 $f = 3.75\ \text{GHz}$
 $\lambda = 80.0\ \text{mm}$
 $Z_L = 30.0 - j\,50.0\ \Omega$
 $d = 1.25\ \lambda = 100.0\ \text{mm}$
 $d = 0$

Set Line

Length units: ☐ [λ] ☒ [m]

Low Loss Approximation

Characteristic Impedance $Z_0 = 50.0\ \Omega$

Frequency $f = 3.75\text{E}9\ \text{Hz}$

Relative Permittivity $\epsilon_r = 1.0$

Line Length $l = 0.1\ \text{[m]}$

Update

$Z_L = 30 + j\, -50\ \Omega$

☒ Impedance ☐ Admittance

Update

Set Generator

$V_g = 1.0 + j\, 0.0\ \text{V}$

$Z_g = 50 + j\, 0.0\ \Omega$

Update

Output

Transmission Line Data 2

SWR = 3.6601 (load)

Amplitude of Incident Voltage Wave [V]
 $V_0^+ = 0.0 - j\,0.5$
= 0.5 $\angle -1.5708\ \text{rad}$

Location of First Voltage Maximum & Minimum
 $d(\text{max}) = 0.38917\ \lambda = 31.1338\ \text{mm}$
 $d(\text{min}) = 0.13917\ \lambda = 11.1338\ \text{mm}$

TIME-AVERAGE POWER
 $P(\text{abs}) = 1.685393\ \text{[mW]}$ Absorbed by load
 $P(Z_g) = 3.820225\ \text{[mW]}$ Absorbed by Z_g

Figure P2.19(b)