2.21 Using a slotted line, the following results were obtained: distance of first minimum from the load = 4 cm; distance of second minimum from the load = 14 cm; voltage standing-wave ratio = 1.5. If the line is lossless and $Z_0 = 50\ \Omega$, find the load impedance.

Solution: Following Example 2.6: Given a lossless line with $Z_0 = 50\ \Omega$, $S = 1.5$, $d_{\min(0)} = 4$ cm, $d_{\min(1)} = 14$ cm. Then

$$d_{\min(1)} - d_{\min(0)} = \frac{\lambda}{2}$$

or

$$\lambda = 2 \times \left(d_{\min(1)} - d_{\min(0)}\right) = 20\ \text{cm}$$

and

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi\ \text{rad/cycle}}{20\ \text{cm/cycle}} = 10\pi\ \text{rad/m}.$$

From this we obtain

$$\begin{aligned}\theta_{\mathrm{r}} = 2\beta d_{\min(n)} - (2n+1)\,\pi\ \text{rad} &= 2 \times 10\pi\ \text{rad/m} \times 0.04\ \text{m} - \pi\ \text{rad} \\ &= -0.2\pi\ \text{rad} = -36.0^\circ.\end{aligned}$$

Also,

$$|\Gamma| = \frac{S-1}{S+1} = \frac{1.5-1}{1.5+1} = 0.2.$$

So

$$Z_{\mathrm{L}} = Z_0 \left(\frac{1+\Gamma}{1-\Gamma}\right) = 50 \left(\frac{1+0.2\exp -j36.0^\circ}{1-0.2\exp -j36.0^\circ}\right) = (67.0 - j16.4)\ \Omega.$$