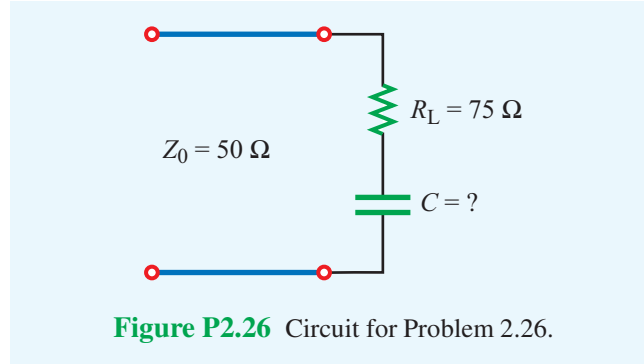


2.26 A $50\ \Omega$ lossless transmission line is connected to a load composed of a $75\ \Omega$ resistor in series with a capacitor of unknown capacitance (Fig. P2.26). If at 10 MHz the voltage standing wave ratio on the line was measured to be 3, determine the capacitance C .



Solution:

$$|\Gamma| = \frac{S-1}{S+1} = \frac{3-1}{3+1} = \frac{2}{4} = 0.5$$

$$Z_L = R_L - jX_C, \quad \text{where } X_C = \frac{1}{\omega C}.$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$|\Gamma|^2 = \left[\left(\frac{Z_L - Z_0}{Z_L + Z_0} \right) \left(\frac{Z_L^* - Z_0}{Z_L^* + Z_0} \right) \right]$$

$$|\Gamma|^2 = \frac{Z_L Z_L^* + Z_0^2 - Z_0(Z_L + Z_L^*)}{Z_L Z_L^* + Z_0^2 + Z_0(Z_L + Z_L^*)}$$

Noting that:

$$Z_L Z_L^* = (R_L - jX_C)(R_L + jX_C) = R_L^2 + X_C^2,$$

$$Z_0(Z_L + Z_L^*) = Z_0(R_L - jX_C + R_L + jX_C) = 2Z_0 R_L,$$

$$|\Gamma|^2 = \frac{R_L^2 + X_C^2 + Z_0^2 - 2Z_0 R_L}{R_L^2 + X_C^2 + Z_0^2 + 2Z_0 R_L}.$$

Upon substituting $|\Gamma_L| = 0.5$, $R_L = 75\ \Omega$, and $Z_0 = 50\ \Omega$, and then solving for X_C , we have

$$X_C = 66.1\ \Omega.$$

Hence

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi \times 10^7 \times 66.1} = 2.41 \times 10^{-10} = 241 \text{ pF.}$$
